CONTINUOUS CURRENT MACHINE DESIGN

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TO THE MEMORY

OF

MY FRIEND AND UNCLE

THOMAS WILLIAM ROBINSON

PREFACE

In attempting to find at a reasonable price, and between one pair of covers, a text-book for the more advanced student of Continuous Current Machine Design, I have met with perpetual difficulty. It seems to me that the text-books hitherto published fall naturally into two groups—those which cost too much, and those which are too special. The former have grown up with the subject, and in process of time successive editions and additions have brought them to a state reminding one of the fifth age of man. To these standard works all students are indebted; though few can afford to buy them: they are indeed often regarded as luxuries to be found in reference libraries only. Books of the second class are usually written to cover special ground at a special time, to act as *vade mecum* to a particular examination, or to render temporary assistance to the expert.

In the present volume I have tried to avoid these extremes by omitting all that a student may reasonably be expected to know at the end of his second year, and by indicating the road along which designers have hitherto travelled and the direction in which the next move is likely to be made. It is my hope that the book may be a useful companion to the student, both in the drawing office and in the class-room, as well as a stimulating and suggestive guide to the teacher. In recent design the introduction of the "commutating pole" or "interpole" has been responsible for a great many changes; and these account for the large proportion of this volume devoted to temperature rise. Yet, lest a change in the price of copper should some day abolish the interpoles, machines not dependent upon them are also fully considered.

Both in the text and in the Examples of Procedure I have tried to lay stress upon the tentative and experimental nature of the

subject, as also upon the number of variables involved, the ever-present influence of cost, and the careful comparisons that should be carried out before any design is adopted. Further, that the text might not be unduly encumbered, those pages which would otherwise look like excerpts from an algebra have been relegated to an appendix. Throughout the calculated examples little or no attempt has been made to carry accuracy further than four significant figures. This is, I think, rational; for the practised designer will be the first to admit that the available data rarely allow of a closer approximation.

It remains for me to acknowledge the substantial assistance I have received from Mr. J. Lustgarten, M.Sc., in Chapters VI., VII., and VIII., and from Mr. Thomas Jones in the diagrams of Chapter XI. To Professor Marcus Hartog, who carefully read the proofs, I am also much indebted, as well as to the various manufacturers who have kindly lent me blocks.

I dare not hope that a book compiled in the spare moments of over-full days can have escaped without some mistakes; if any reader will bring such errors to my notice I shall be very grateful.

WILLIAM CRAMP.

20, Mount St., Manchester,
August, 1910.

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LIST OF SYMBOLS

NOTE.—Wherever the use of symbols could be avoided without objectionable complication of the printing, this course has been followed ; and even when symbols are used in the text their meaning is often repeated, so as to reduce the necessity for reference to the list below. The symbols used in the text are all given here, but in the Appendices it has been necessary to make use of a few more, which are carefully explained where they occur. All dimensions are in English measure.

A with suffix, as A_m (p. 75). The area of a particular part or section in square inches.

a A constant in the temperature formula (p. 80) (except in Fig. 26).

B Bending moment in inch-lbs. (p. 157).

b A constant in the temperature formula (p. 80) (except in Figs. 26 and 30).

b_1 Width of spider spoke parallel to shaft (p. 161).

C The total current in amperes taken from or given to a machine (p. 128).

C_h The heating coefficient of magnet coils (p. 70).

C_w The current in any armature conductor in amperes.

D The outside diameter of the armature in inches.

d The diameter of the pole-core, or, in the case of square poles, the length of a side of the square, in inches.

d_c The depth of a coil in inches (p. 73, Fig. 44).

E E.M.F. in volts (p. 128).

e The commutating E.M.F. in volts (p. 121).

f The flux set up in one turn of the coil by one ampere flowing therein (p. 122).

g The number of coils short circuited by a brush (p. 122) (except Fig. 26).

K Used generally for various constants.

h_1 Thickness of spider spoke at its smallest section (p. 161).

L The gross length of an armature core in inches.

L_c The coefficient of self-induction of an armature coil under commutation (p. 122).

L_e Length of armature over end connections (p. 162).

l The net length in inches of an armature core.

l_c Length of field coil (p. 73, Fig. 44).

m The number of armature windings (p. 99).

m_1 The ratio width of slot : width of tooth at the armature periphery (p. 15).

m_2 The fringing factor (p. 15).

N The flux per pole in C.G.S. lines (p. 17).

n The revolutions per second of the armature.

P (with different suffixes). Lost power in watts (pp. 72 and 80).

p The number of poles.

q Ampere-conductors per inch of armature periphery (p. 22).

R (with different suffixes). Resistance in ohms (pp. 26 and 27).

r Resistance of the circuit of a coil under commutation.

S The number of sections in a commutator.
T° Temperature in degrees Centigrade.
T On pages 64 and 65 the time constant, and on p. 157 twisting moments.
t Instantaneous values of time (p. 65).
t_c The time of commutation in seconds (p. 118).
V Special values of E.M.F. (pp. 116 and 125).
V_h Reactance voltage according to Hobart (p. 127).
V_r Linear reactance voltage (p. 127).
v Linear circumferential velocity of the armature in feet per minute (p. 80).
w Total number of armature conductors.
X The "electric loading" of the armature (p. 21).
x Reactance in ohms of a coil under commutation (p. 127).
Y Magnetic loading of the armature (p. 21).
y The number pitch of an armature winding (p. 94).
y_f Forward number pitch (p. 94).
y_b Backward number pitch (p. 94).

GREEK LETTERS.

α Modulus of elasticity (p. 157); also as an angle (p. 162), as a temperature coefficient of copper (p. 67), and as cross-section of a conductor (p. 96).
β (with various suffixes). Density in lines per square inch (p. 38).
ϵ Base of the natural logarithms (p. 121).
η (with various suffixes). Efficiency (pp. 26 and 27).
θ Temperature in degrees Centigrade (p. 65) in particular cases.
λ Leakage factor (p. 44).
ρ Specific resistance of copper.

ABBREVIATIONS.

L.M.T (with various suffixes). Length of mean turn.
t.p.s. Armature turns per commutator section.
K.W. Kilowatts.
H.P. Horsepower.
~ Frequency in cycles per second.
S.W.G. Standard wire gauge.
A.T. Ampere-turns.
D.C.C. Double cotton covered.
S.C.C. Single cotton covered.
P.$_p$. Pole-pitch.

ERRATA

Page 13, line 28, for 3000 read 30,000.

" 21, Table I., for $\frac{6}{5}$ read $\frac{5}{6}$.

" 22, for $\frac{D^3\lambda}{P}$ read $\frac{D^3}{P}$.

" 26, line 2, for "shunt" read "shunt or compound."

" 43, Formula (*b*), for 7D read 0·7D.

" 43, " (*c*), for 5·3D read 0·53D.

Pages 47 and 48, for 6986 read 6786.

Page 60, line 26, "the armature of a dynamo."

" 61, line 25, for "15" read "13."

" 76, equation (2), for 285 read 275, and multiply the expression by $\frac{4}{\pi}$ (cf. p. 230).

" 77, line 9 and equation (2), for 142·5 read 137·5.

" 77, equation (2), multiply by $\frac{4}{\pi}$ (cf. p. 230).

" 82, line 33, for 42 read 47.

" 112, line 1, for "it is 5" read "it is 6."

" 112, line 2, for $(y_f - 1)$ read $(y_f \pm 1)$.

" 127, line 32, for (*t.p.s.*) read $(t.p.s.)^2$.

" " fig. 118, "scale 1″ = 1′" delete.

" 176, line 38, for "ampere-turns per pole" read "flux per pole."

" 208, Equation (1), read $450 = 4{\cdot}7d_c^2 + 47(l_c + d_c) + 6{\cdot}28l_cd_c$.

" 223, read $w_s + w_t = \pi D/t$.

" 233, line 16, for "turns" read "lines."

CONTINUOUS CURRENT MACHINE DESIGN

CHAPTER I

THE FORM OF MODERN MACHINES

Introduction.—All continuous-current machines, whether motors or dynamos, consist of three main parts, namely—

(1) The field-magnet.
(2) The armature.
(3) The collecting gear.

It is the business of the designer so to construct and proportion these parts as to obtain the best results for a given expenditure of capital outlay as well as of energy; further, it is his work to fit them together and to arrange them in their proper relative position by means of suitable mechanical devices.

In considering, therefore, the design of continuous-current machines, we shall, first of all, set forth the laws governing the proportions of the main parts, and later give some examples of the design of the main mechanical supports and attachments. It is impossible to separate entirely the electrical and magnetic design from that which is purely mechanical; for very often it happens that the one partly determines or limits the other, and such instances will be brought out as the work proceeds.

General Forms.—Now, because the function of a well-designed magnetic circuit is to allow of the existence of a magnetic flux with as little reluctance as possible, the modern machine is arranged in one of the various forms depicted in Figs. 1 to 4. The most important of these is the shape shown in Figs. 1 and 2, and *the remarks which follow will always be made with reference to such a machine unless the contrary is definitely stated.*

In the case of small machines, the over-all dimensions and the proportions for a given output are dictated by questions both of *economy* and *appearance,* while in the larger sizes economy is usually the ruling factor.

Present Practice.—Whatever machine we examine we find that, as the field-magnet shape has become practically standardized to the form shown in Figs. 1 or 2, so the armature is practically standardized

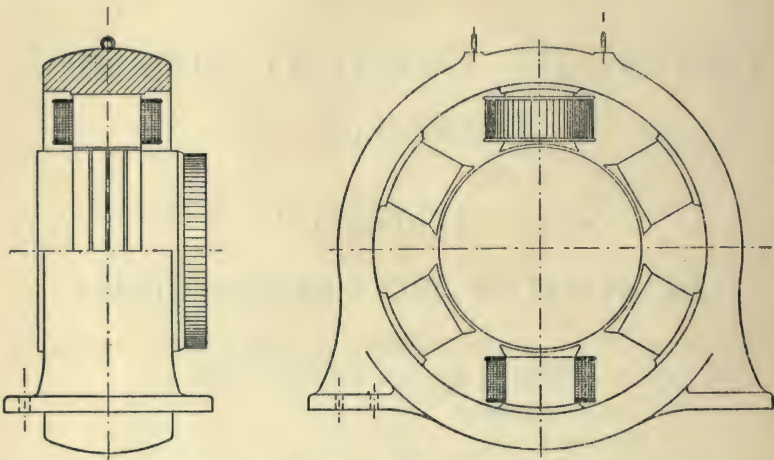


FIG. 1.—STANDARD MULTIPOLAR FIELD-MAGNET WITH ARMATURE.

to the general shape of Fig. 3, while the commutator is as shown in Figs. 3 and 4.

Form of Field-magnets.—The form, however, of all these parts is affected by the material to be used for their manufacture. In the case of field-magnets the materials adopted are cast iron, wrought iron, mild cast steel, and malleable iron.

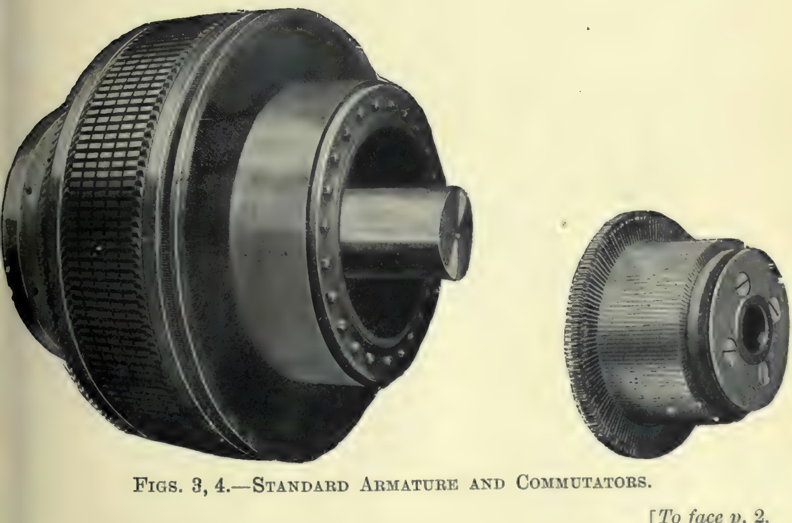
Cast Iron is a material of low magnetic permeability, but also of low cost.

Cast Steel and **Wrought Iron** are comparable in respect both of cost and of permeability.

Some relative values of permeability in the respective cases are shown by the curves given in Figs. 5 and 6, which are obtained from average samples of the different materials; from these it will be seen (since the cost of cast iron is about $\frac{3}{5}$ that of cast steel, while the relative induction of the former to the latter for a given number of ampere-turns is about as 5 is to 3) that for cases like the yokes of field-magnets, where no copper surrounds the iron, the question of material resolves itself into one of weight and mechanical strength, and not of cost.

Those parts of the magnet-frame which are surrounded by copper must be so designed as to carry the maximum flux while having the minimum perimeter, *i.e.* they must be made of material of the highest permeability; and also they must be of such a shape

Fig. 2.—Arrangement of Field-Magnet with Interpoles.



Figs. 3, 4.—Standard Armature and Commutators.

[To face p. 2.

as to give maximum sectional area with minimum perimeter, *i.e.* their section should be nearly circular. Thus, except in special instances, cast iron is inadmissible for field-poles.

Wrought Iron is a material of high permeability, but it does not lend itself to special shapes. Circular poles are rarely made of wrought iron; but this material has the great advantage that it can

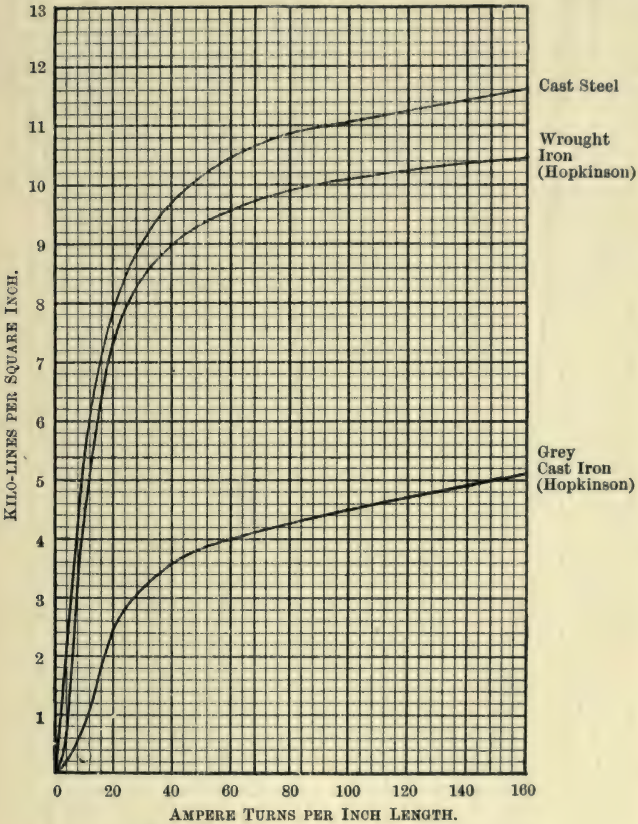
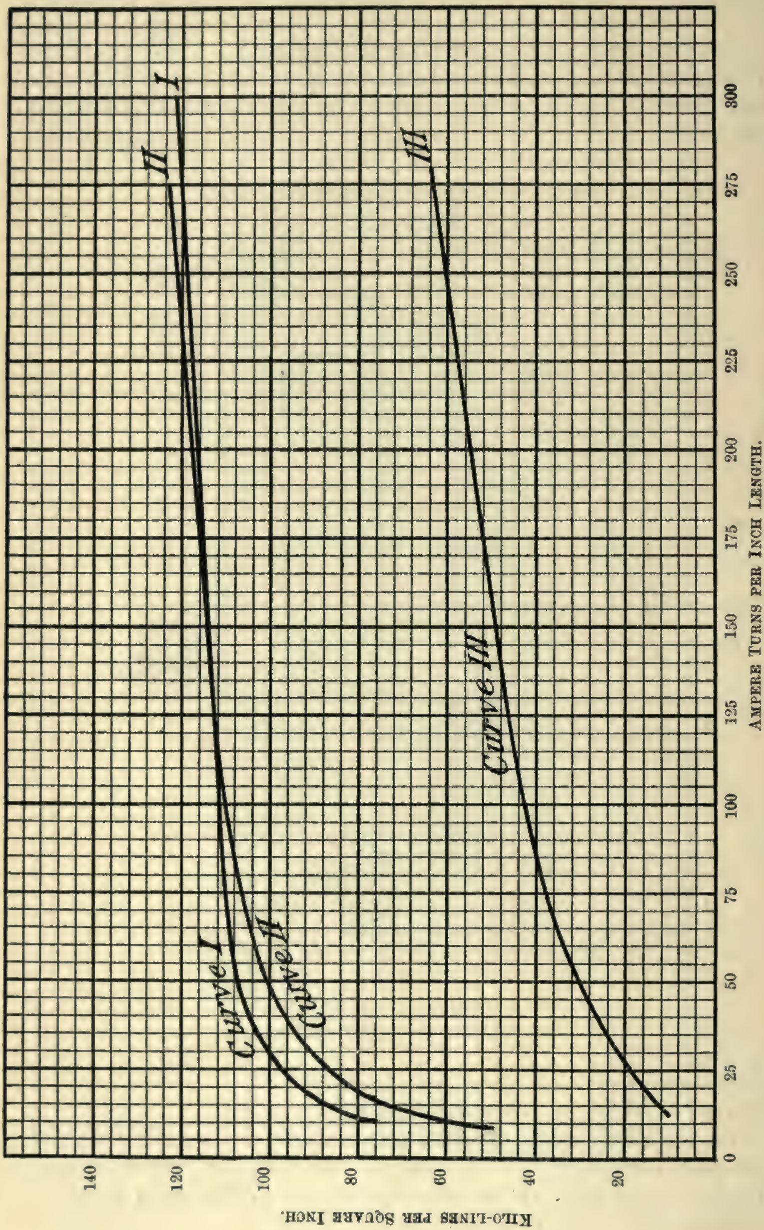


Fig. 5.—Curves of Permeability.

easily be obtained in thin sheets, or laminæ, and in this form it is usually used to build up rectangular or square poles.

Where it is desired to cut down to a minimum the eddy-currents in poles and pole-shoes, these laminæ are adopted either for the complete pole and shoe or for the shoe alone. Examples of these constructions will be given later.



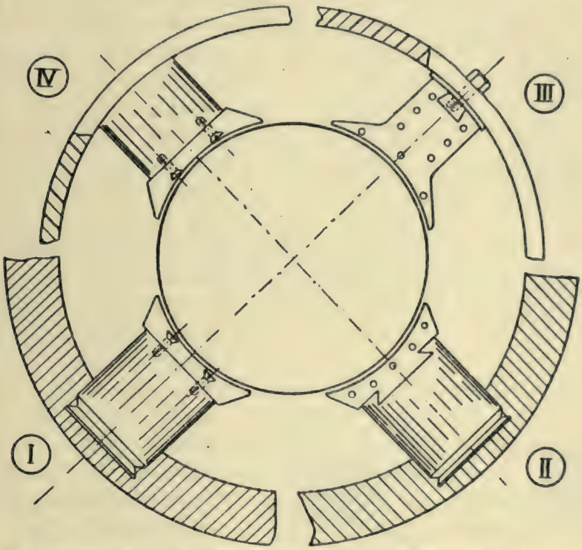
CURVE I. Sankey—Wrought-Iron Armature Stampings.
CURVE II. Edgar Allen—Steel.
CURVE III. Good Cast Iron.

FIG. 6.—SATURATION CURVES.

Cast Steel has the two advantages of possessing high permeability and of being easily cast; it is therefore very suitable for magnet frames made complete with their poles, the latter being then generally circular or oval in section.

Malleable Iron has a permeability midway between cast iron and cast steel; but its cost is practically equal to that of the latter, and its only compensating advantage is that often readier delivery can be obtained, especially for small or complicated shapes. It is therefore only used where urgency dictates.

Field-poles.—In the construction and fixing of field-poles, there is a choice of several methods. The pole may be cast with the yoke,



I. Mild Steel Pole cast into Cast-Iron Yoke and fitted with Shoe.
II. Mild Steel Pole cast into Cast-Iron Yoke and fitted with Laminated Shoe.
III. Laminated Pole and Shoe fitted to Cast-Steel Yoke.
IV. Pole and Yoke made in Cast Steel and Shoe fitted as in I.

FIG. 7.—METHODS OF FIELD CONSTRUCTION.

in which case the poles must be made either of cast steel or of malleable iron if the field copper is to be reduced to a minimum; or the steel pole may be fixed separately, in which case there is an extra surface to machine; or stampings may be riveted together and fixed into the yoke when the section or shape of the pole is limited, as mentioned above.

Examples of various constructions are seen at I, II, III, IV, Fig. 7.

In all cases, because the pole must be small (to keep down the amount of copper), a shoe must be used to reduce the air-gap density;

and it is in satisfying these two conditions that the high-permeability pole is of such use and importance.

Machines without Pole-tips.—All sorts of constructions have been used to obviate the necessity for the pole-tip; thus, for instance, one maker casts pole, shoe, and yoke all in one. He then gets his field-coil on by having the tip all on one side of the pole (Fig. 8). Similarly, some makers cast the pole hollow, and put the large field-coil on as shown in Fig. 9.* In each case the object is to get rid of the joints in the frame, and at the same time to keep down the weight of field-copper. As regards ordinary methods, the four illustrated in Fig. 10 are typical; and from these it is seen that a butt joint may be arranged either between pole and shoe, or pole and yoke, but it is advisable not to have butt joints at *both* these places; for the effect of joints in the magnetic circuit is always to increase the reluctance of the path, this increase depending upon the badness of the fit. Even with an apparently perfectly fitted joint, an additional reluctance is introduced, which has been estimated as corresponding to an air-gap of about 0·0005 inch; and in addition to this, the leakage factor is somewhat increased.

Fig. 8.—Arrangement of Tip to avoid Magnetic Joints.

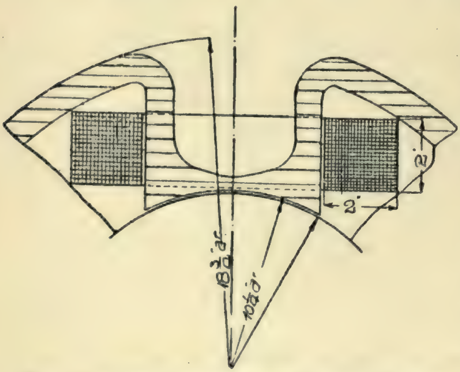


Fig. 9.—Arrangement of Hollow Pole.

Ordinary Forms of Field-construction.—Usually the form that pays best is that with round pole cast into a C.I. yoke, as shown at I, Fig. 7, but naturally this depends to some extent on the works and on the tools available.

The writer prefers to use a cast steel or mild steel pole as often as possible, believing that to be the most economical arrangement, especially in small machines. The length of the armature-stampings is therefore limited, where possible, to allow the use of a round steel pole.

* See "Magnetic Leakage and its Effect in Electrical Design," *Proc. Inst. Elec. Eng.*, vol. **38**, No. 183.

Laminated Poles.—We have previously seen that laminated poles must be rectangular in section, and that the coil around that rectangular section needs a greater amount of copper than would with circular poles be necessary. But there is a corresponding advantage in the reduction of eddy-currents in the poles themselves. The effect of the iron-losses (from eddy-currents and hysteresis) in continuous-current machines becomes more and more obvious every day; and the reason for this is that makers attempt to use armature discs with fewer and fewer slots in them. The fewer the slots the less is the number of insulating tubes, and therefore the higher is the space-factor for a given winding. The fewer the slots, the greater the difference produced in the change of flux between the teeth and the slots; so that, whereas with a large number of slots we have almost a uniformly distributed flux, in the case of few slots we have the flux lines in large bunches, alternating with little spaces almost free from flux. It is this unequal distribution of flux which tends to set up eddy-currents in the pole, and which may be reduced by means of a properly laminated pole. For most cases the laminated shoe practically does away with all the eddy-currents, and there is no sensible heating in the poles except in the case of traction-motors, where laminated poles are almost a necessity.

Comparison of Circular and Rectangular Pole.—In order to bring out the advantage of a circular pole, let us compare the costs of copper for a circular pole and for a square pole of the same sectional area, say for the following conditions:—

The resistance per coil in each case is to be about 180 ohms, while the voltage per coil is 100, and the ampere turns 4000. We will adopt a pole of 50 square inches cross-section; then in the case of the circular pole the diameter will be 8 inches.

Circular Pole.—Now, for a coil of this resistance a winding depth of about $1\frac{1}{2}''$ is required (Chap. VI.).

$$\text{Length of mean turn then} = 29{\cdot}85''$$

$$\text{resistance of mean turn} = \tfrac{100}{4000} = 0{\cdot}025 \text{ ohm}$$

$$\text{and resistance per 1000 yards} = \frac{36 \times 1000 \times 0{\cdot}025}{29{\cdot}85} = 30{\cdot}15 \text{ ohms}$$

From the wire tables we find that the resistance per 1000 yards of No. 21 S.W.G. is 29·9 ohms cold, or about 33 ohms when warm, this being the nearest size to the above requirement.

Hence, with this wire, which has a diameter covered of $0{\cdot}042''$—

$$\text{Turns per layer} = \frac{8}{0{\cdot}042} = 190$$

$$\text{Layers per coil} = \frac{1{\cdot}5}{0{\cdot}042} = 35$$

$$\text{Turns per coil} = 190 \times 35 = 6650$$

$$\text{Resistance per coil} = \frac{6650 \times 29{\cdot}85 \times 33}{36 \times 1000} = 182 \text{ ohms}$$

From the table we find that No. 21 S.W.G. weighs 9·301 lbs. per 1000 yards, so that—

$$\text{Weight of copper per coil} = \frac{6650 \times 29{\cdot}85 \times 9{\cdot}301}{36 \times 1000} = 51{\cdot}3 \text{ lbs.}$$

Square Pole.—If we adopt the square pole, the winding depth to fulfil the same requirements will be greater—say 2″.

Length of mean turn = 36·28″ (the dimensions of the pole being 7·07″ × 7·07″).

$$\text{Resistance per 1000 yards} = \frac{36 \times 1000 \times 0{\cdot}025}{36{\cdot}28} = 24{\cdot}8 \text{ ohms}$$

The nearest wire is No. 20 S.W.G., which has a resistance of 23·6 ohms per 1000 yards cold, *i.e.* 26 ohms warm.

Hence, with this wire, which has a covered diameter of 0·046″—

$$\text{Turns per layer} = \frac{8''}{0{\cdot}046''} = 174$$

$$\text{Layers per coil} = \frac{2''}{0{\cdot}046''} = 45$$

$$\text{Turns per coil} = 174 \times 45 = 7830$$

$$\text{Resistance per coil} = \frac{7830 \times 26 \times 36{\cdot}28}{36 \times 1000} = 205 \text{ ohms}$$

From the tables, No. 20 weighs 11·77 lbs. per 1000 yards.

$$\text{Weight of copper per coil} = \frac{7830 \times 11{\cdot}77 \times 36{\cdot}28}{36 \times 1000} = 91{\cdot}3 \text{ lbs.}$$

So that, although the resistances of these coils only differ by about 12 per cent., the weight of copper for the square pole is 78 per cent. greater than that for the circular pole, and putting the value of copper and labour at 1*s.* 6*d.* per pound of material, we effect a saving of £3 per coil by adopting the circular section rather than the square. Comparisons between circular and rectangular sections show more and more to the advantage of the former as the rectangle departs further and further from the square. Thus a circular section is most economical; and of rectangles the square is best.

It would seem that many designers hardly realize the bearing of this calculation, which is of special importance in the case of small machines, where the weight of field copper is to armature copper in the proportion of about 4 to 1. This ratio decreases very considerably in large machines, so that the question of circular as against rectangular field-poles, requires more careful consideration, especially as the round pole will often give insufficient cooling surface. These calculations are affected also to a certain extent by the question of

leakage factor; where the latter amounts to 1·4, it may in certain instances pay to design a field quite differently.*

Other Types of Field-magnet. — The question of the shape of field-magnets cannot be dismissed without a reference to three other types. The first is that known as the "Lahmeyer"; it is depicted in Fig. 10, and is the most economical form of bipolar machine yet designed. Fashion, however, is against it, for it is practically impossible now to sell a machine which has other than a circular yoke.

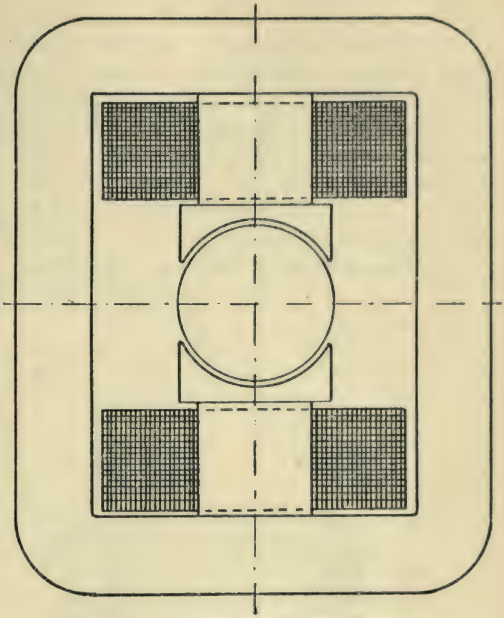


FIG. 10.—LAHMEYER FIELD MAGNET.

The second type which deserves consideration is known as the "Manchester" (Fig. 11), and it has been able to hold its own on account of its extreme steadiness, due to the low position of the armature. It

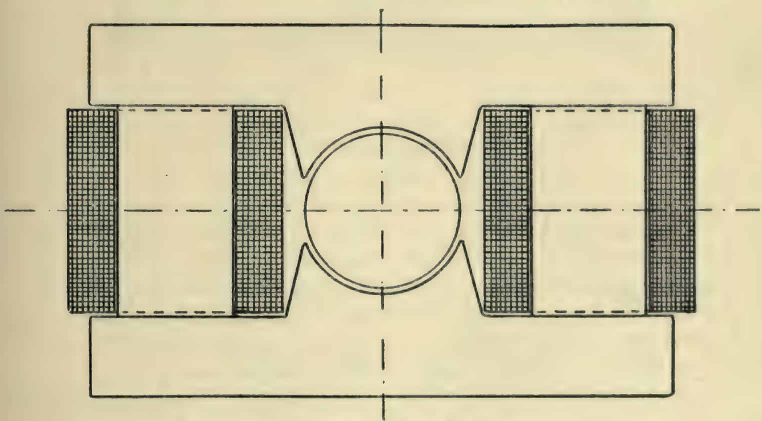


FIG. 11.—"MANCHESTER" FIELD MAGNET.

* See the author's paper on "Magnetic Leakage and its Effect on Electrical Design," *Jour. I.E.E.*, vol. **38**, pp. 548 *et seq.*

possesses, however, the two disadvantages of large magnetic leakage and many joints in the frame.

The third type is the old bipolar machine (Fig. 12), which has some curious advantages. In the modern machine, with the poles fixed radially, the area of the gap cannot be increased beyond the dimensions of the pole unless a pole-shoe be put on. In this bipolar case, however, the pole-arc is largely independent of the section of the pole, so that almost any area of air-gap may be obtained without the use of a shoe and without increasing the pole sectional area; that is to say, with reasonable air-gap or pole densities we may still slip the finished field-coils over the ends of the magnet.

Multipolar and Bipolar Machines.—The foregoing machines

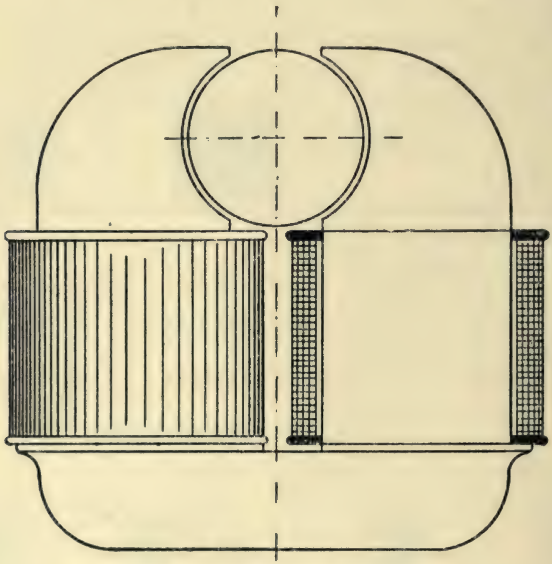


FIG. 12.—OVERTYPE MAGNET.

may be summarized as follows: *Were it possible always to construct bipolar machines, the most economical dynamo would usually be found to be one having circular poles and an armature whose length was as great compared to its diameter as that field shape would allow.*

In support of this statement, we have to consider the following factors as affecting the economy of field-magnets:—

(1) *The weight of copper on the field-magnet*, which is determined by—

(*a*) the ampere-turns per pole;
(*b*) the number of poles;

(*c*) the length of mean turn per coil; and
(*d*) the watts lost.

Of these—

(*a*) The ampere-turns per coil are dependent chiefly upon the air-gap length and the tooth saturation.
(*c*) The length of mean turn per coil depends largely upon the pole perimeter.

(2) *The weight of steel for the poles* depends for a given material upon the flux-density in the pole, the number of poles, and the sectional area per pole, *i.e.* upon the total sectional area of all the poles, if the total flux and the material be known.

Now, the air-gap length is decided largely by mechanical clearance, and other considerations which are practically independent of the number of poles. Whence, the number of ampere-turns per pole is practically independent of the number of poles.

It is easy to see that the geometrical figure which has the greatest sectional area for least perimeter is the circle; so that a comparison of the economy of bipolar and multipolar fields may be made on the assumption of constant total flux; watts lost and material being the same in both cases. Therefore the comparison is really between an area made up of a number of small circles (multipolar) as against one large circle, *i.e.* between the perimeter of a circle of a sectional area S, and the sum of the perimeters of n small circles each of area $= \frac{S}{n}$, which are in the ratio of $1 : \sqrt{n}$. Obviously the comparison is immensely in favour of the bipolar case.

It is not, however, possible to construct large machines of bipolar form, because of the difficulties of commutation and the large magnetic leakage. On the other hand, small machines which are bipolar are not as economical as they might be, on account of the necessity for the circular yoke imposed by public taste.

What has been said above applies equally to machines with one field coil, like the Lundell motor, for a full discussion of which the reader is referred again to the *Proc. I.E.E.*, vol. **38**, p. 558.

General Form of Armatures.—Practically only one material, viz. soft iron or very mild steel, is available for armature cores; and nowadays there is but one shape of core in use, viz. the slotted drum. Further, for both small and large machines the "barrel" type of winding is now adopted (Fig. 3), so that unless some other form is specially mentioned, the constants, etc., given in this book will refer to machines having armatures exactly in accordance with this description.

General Form of Commutators.—Nearly always the commutator

is of the same general type (Figs. 3, 4, and 111), and this is characterized by the end-ring, having a small angle (of about 5°) on the upper surface compared with the angle on the lower side of between 30° to 40° (see p. 165). Some designers prefer to keep the upper side horizontal. The dimensions are almost standardized now, so that for a given current density at the brush and a given speed, there is a constant temperature rise; and two machines of similar output, although of different manufacture, will almost always have commutators of the same cylindrical surface.

If it could be proved to-morrow that copper was not the best material for commutator segments (and it is very doubtful whether it is), then the form and dimensions of commutators would change immediately.

CHAPTER II

GENERAL PROPORTIONS OF MODERN MACHINES

As has been shown, choice of material affects the question of general form; it also dictates the general proportions.

Having already practically decided on the material for the poles, the armature, and in some cases the yoke, we find that the general proportions are ruled by a number of factors. In the case of the field-magnet, the general determining factors are—

(1) The maximum densities permissible, and the ratio of pole-arc to pole-pitch.

(2) The watts to be dissipated in the field-coils, and the temperature-rise of the latter.

(2) will be dealt with under the heading of "Temperature-Rise."

From very careful comparison of a number of designs, certain densities have now been adopted for continuous-current machines which are employed by nearly all modern designers.

Limiting Densities: (1) Yoke.—If cast iron be used, the density varies from 26,000 to 45,000 lines per square inch. (It must be very soft cast iron for the number of lines to exceed 45,000.) A density of from 45,000 to 90,000 lines per square inch may be used if the yoke be of cast steel.

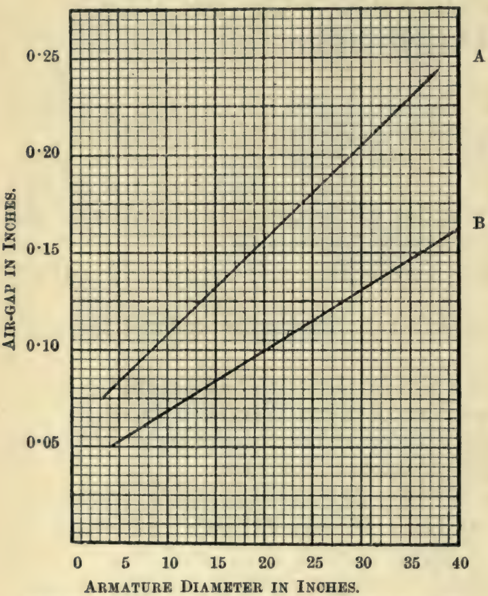
(2) **Pole.**—Passing on to the pole (which will be of wrought iron or mild steel), the density at full load should not exceed 120,000 lines per square inch, and should for safety be less; a very fair value being 100,000.

(3) **Air-gap.**—Here the densities used are somewhat more vague, and depend very largely upon the size of the machine. In a small motor, say 5 H.P., the density in the air-gap is about 45,000 lines per square inch, and even less. In still smaller motors of 1 H.P. or less, ~~3000~~ lines per square inch is the maximum density admissible. In large generators (500 K.W.) 60,000 is a common figure.

Air-gap Length.—The air-gap length cannot be reduced in proportion to the armature diameter; it must not be less than $\frac{1}{16}''$, nor need it be more than $\frac{3}{8}''$ from the point of view of mechanical clearance.

The modern tendency is to keep down the air-gap in larger machines, and to this end interpoles contribute, as is evidenced by Fig. 13, which gives average values of the air-gap in modern practice.

Relative Constancy of the Field Ampere-turns.—In Fig. 13 have been given the usual modern air-gap lengths for various sizes of machine. If these be multiplied by 0·313 and by the air-gap densities given above, we have the gap ampere-turns per pole for each size of machine. Similarly, the ampere-turns for the armature teeth may be approximately predetermined, and the sum of these two is usually 80–85 per cent. of the field ampere-turns per pole. It is clear from the air-gap curves that the length of gap increases very slowly with increase of output, and in consequence the ampere-turns per pole are relatively constant.



A without interpoles; equation $y = \cdot 06 + \cdot 00487\,x$ nearly.
B with interpoles; equation $y = \cdot 036 + \cdot 00316\,x$ nearly.

FIG. 13.—AVERAGE VALUES OF AIR-GAP LENGTHS.

The Ratio Pole-arc : Pole-pitch.—The next factor that affects the proportions is the ratio of pole-arc to pole-pitch, which is determined by two considerations, namely—

(*a*) Relationship between ampere-turns of the armature and ampere-turns of the field; and

(*b*) Neutral zone required to give good commutation.

Modern designers vary considerably in their choice of this ratio, as from 0·6 to 0·85. A good average value frequently adopted is 0·7. The following considerations will influence the choice of the ratio:—

In small machines a large ratio means a large leakage-factor, which is undesirable; further, the weight of copper on the field being so much in excess of that on the armature, it is not infrequently possible to cheapen a machine by increasing the diameter of the armature without any alteration to the poles, *i.e.* by simply altering the ratio pole-arc to pole-pitch. To decide this point it is absolutely necessary to take a mean ratio, such as 0·7, and from a trial design to estimate roughly

the cost (see Chap. XII.); then by changing the ratio the cost may be estimated anew.

Armature-teeth.—Passing now to the question of armature-teeth. For a given armature-diameter, the greater the density in the armature-teeth the more room is occupied by copper; therefore, the greater the density in the teeth the higher is the output of a machine for a given diameter of armature. Seven or eight years ago designers advised a maximum density at the roots of the teeth (on the assumption that all the flux passed through the teeth) of 118,000 lines per square inch, while to-day this is sometimes as high as 195,000. It is advisable to keep the apparent maximum density in the teeth between 120,000 and 160,000 lines per square inch, because there is little advantage in forcing up this density beyond the point where the curve for the iron is parallel to that for an air-gap.

Now, if we know the ratio of the density at the pole-face to that at the roots of the teeth, also the ratio gross to nett length of armature-core, then for given ratio width-of-slot to width-of-tooth the depth of the tooth is determined in terms of the armature-diameter, thus: *

$$\text{Depth of tooth} = \frac{1 - \mathrm{K}(1 + m_1)}{2(1 + m_1)}\mathrm{D}, \text{ where}$$

$$\mathrm{K} = \frac{\text{density at pole-face}}{\text{apparent density at root of teeth}} \times \frac{\text{gross length of armature}}{\text{net length of armature}} \times \frac{1}{m_2}$$

m_2 being a factor which allows for fringing at the pole-tips, and

$$m_1 = \frac{\text{width of slot}}{\text{width of tooth at armature-face}}$$

The following are some very general values for the different terms in the above equations :—

$$\left.\begin{array}{l}\text{Density of pole-face} = 54{,}000 \text{ lines per sq. in.}\\ \text{Density of root of teeth} = 146{,}000 \quad ,, \quad ,, \quad ,,\end{array}\right\} \textit{i.e.}\ \text{ratio} = 1 : 2{\cdot}7$$

Ratio of net length to gross length of armature = 0·8

$m_2 = 1{\cdot}1$ $\qquad$ $m_1 = 1$

and width of slot = width of tooth at face

With these values depth of tooth = 0·04D *

Of these values, those given for the pole-face and teeth cannot change much. The ratio $\frac{\text{nett}}{\text{gross}}$ length is almost fixed, as also is m_2; m_1 † has been shown by Professor S. P. Thompson to be practically best as unity or thereabouts; the absolute best ratio seems to be such that the width of copper in the slot is equal to the width of tooth-root, which however is also in practical agreement with $m_1 = 1$.

Number of Teeth.—The number of teeth to be adopted is

* See Appendix I.

† See *Journal Soc. Arts*, vol. 54, p. 997.

affected chiefly by two considerations, viz. that economy dictates as few teeth as possible, while perfect flux-distribution and commutation demand a large number of teeth.

It has been shown that the depth of the slots depends chiefly upon the armature diameter when the ratio width of tooth to width of slot is fixed.* From these two factors the total aggregate slot area is settled, and the smaller the number of teeth, the less is the room taken up by insulation.

On the other hand, the greater the number of coils per slot, the greater is the difference between the position of the slot (with respect to the pole-shoe) when the first coil is under the brush, and its position when the last coil is commuted. Now, this difference must not be too great (particularly if the ratio $\frac{\text{pole arc}}{\text{pole pitch}}$ is small), if good commutation is to be obtained. So that a practical limit to the coils per slot is set, and this varies between one and ten. This question must be discussed more fully under the heading of Commutation, but a compromise of this nature can only be determined by practice, so that an empirical rule for the number of slots for estimating purposes will not be out of place here. Generally speaking, a good first approximation is arrived at by making—

The number of slots equal to four times the armature diameter in inches.

Another consideration affecting the number of slots slightly is the *type of winding* adopted. This will be more particularly dealt with under "Armature Windings" (Chap. VIII.).

Dimensions of the Armature below the Teeth.—In small machines a density as high as 95,000 lines per square inch may be used, and this determines the size of stamping. In large machines, the depth of the stamping is ruled very often by considerations quite apart from magnetic density.

For if calculated for a density as high as 100,000 in a slow-speed machine, the stampings would be mechanically too weak. Further, necessity for ample internal ventilation will often dictate the best density, and the question of permissible iron loss affects the matter to some extent (see p. 221).

Recently, some new magnetic materials combining high permeability with low iron-loss have been brought into the market. Of these the best known is "stalloy," whose curve is given plotted with that of "lohys" (an excellent soft iron for armature sheets) in Fig. 14. The comparative iron-losses for plates 0·02″ thick are shown in Fig. 16, and the actual iron-losses for stalloy in Fig. 15.

It is a curious fact that for continuous-current machines not all

* See Appendix I.

the apparent advantage of stalloy can be obtained. The reason for this is not known; yet the fact is sufficiently marked to lead makers to adopt stalloy for alternating-current apparatus, and to adhere to brands such as lohys for direct-current machines.

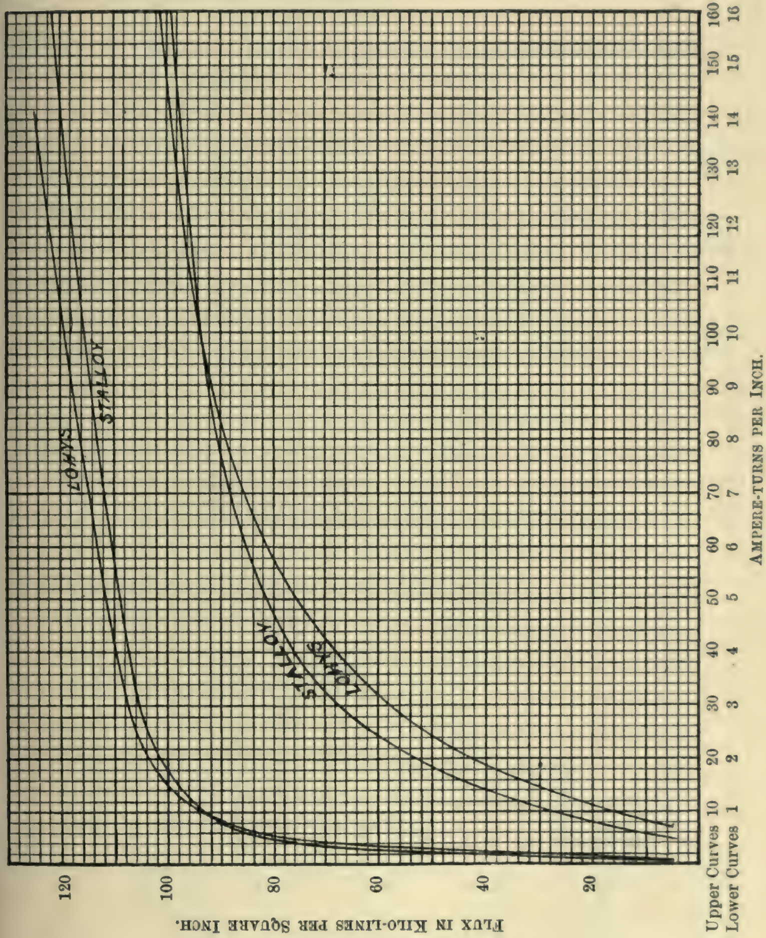


FIG. 14.

Apart from these modifying factors, the density aimed at below the armature-teeth should be about 80,000 lines per square inch.

Once having fixed the flux per pole N, then all the main dimensions of the machine are practically determined.

$$\text{For the area of cast-iron yoke} = \frac{N}{2 \times 40{,}000} \text{ about}$$

$$\text{Area of pole} = \frac{N}{100{,}000} \text{ about}$$

$$= \pi \frac{(\text{dia. pole})^2}{4}$$

$$\text{or for square poles} = (\text{pole side})^2$$

$$\text{Area of shoe} = \frac{N}{54{,}000} = \left\{ \begin{array}{l} \text{axial length of} \\ \text{shoe} \times \text{pole-arc.} \end{array} \right.$$

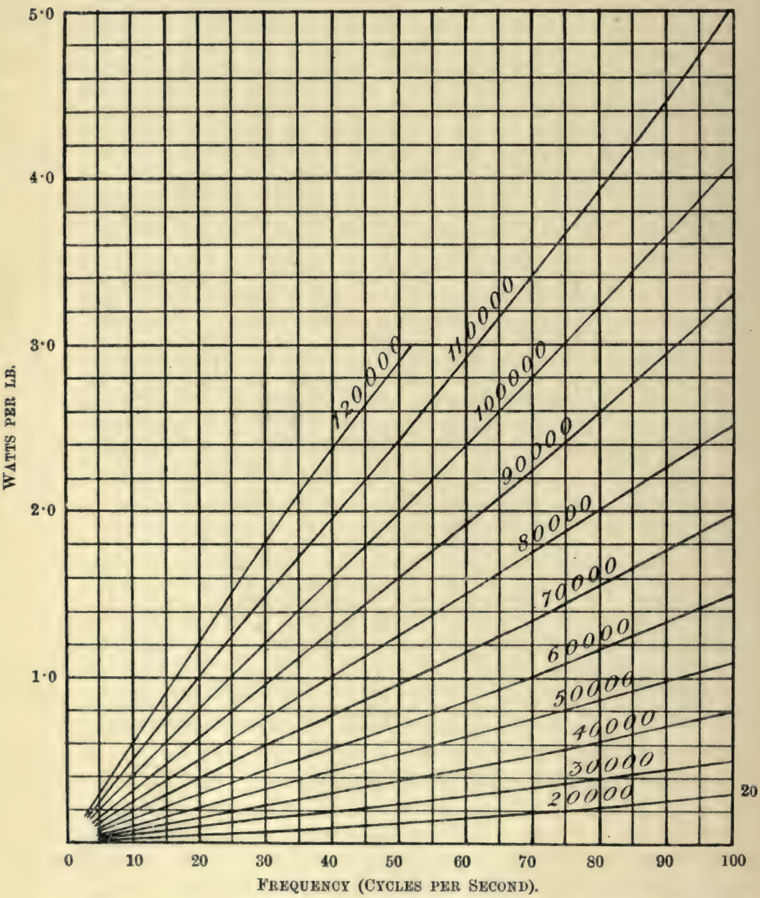
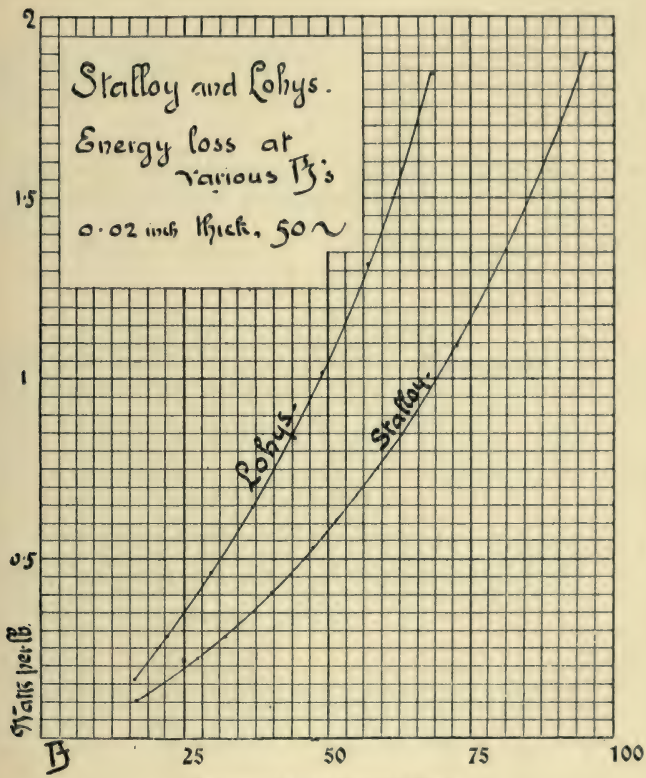


FIG. 15.—STALLOY HYSTERESIS AND EDDY-CURRENT LOSSES FOR VARIOUS DENSITIES IN LINES PER SQUARE INCH.

And since $\frac{\text{pole-arc}}{\text{pole-pitch}} = 0{\cdot}7$ generally, and the axial length of the shoe cannot differ greatly from the pole-diameter or pole-side, we get—

$$\frac{N}{54{,}000} = \left\{\begin{matrix}\text{pole-diameter or}\\ \text{pole-side}\end{matrix}\right\} \times \text{pole-pitch} \times 0{\cdot}7.$$



KILO-LINES PER SQUARE INCH.

FIG. 16.

The art of designing then resolves itself into adapting the most economical flux to the greatest number of conductors on the armature.

Relationship between Field and Armature Dimensions.—We have seen that the most generally economical field for machines under 100 K.W. is one with a round pole, although a square pole will work out well. It is often difficult to decide the number of poles for a given output; but leaving for the moment the number of

poles per K.W., an important relationship may be established between the number of poles of given diameter, and the diameter of the armature. The length of armature will be approx. = diameter of pole, but it may be a little longer, to leave a ledge for the support of the field-coil.

$$\text{Pole-pitch} = \frac{\pi D^*}{p}$$

$$\text{Pole-arc} = \frac{\pi D}{p} \times 0{\cdot}7 \text{ (allowing ratio } \frac{\text{pole-arc}}{\text{pole-pitch}} = 0{\cdot}7\text{)}$$

$$\therefore \text{ area of pole-shoe} = L \times \frac{\pi D}{p} \times 0{\cdot}7$$

The density at the pole-face must lie somewhere between 50,000 and 60,000 per square inch, taking 54,000 as a first approximation.

$$\text{Total flux per pole} = L \times \frac{\pi . D}{p} \times 0{\cdot}7 \times 54{,}000$$

$$\text{Area of pole} = \frac{\pi d^2}{4}$$

$$\text{Flux per pole} = \frac{\pi d^2}{4} \times 10^5 \left\{ \begin{array}{l} \text{(taking } 10^5 \text{ as the density} \\ \text{per sq. inch in pole)} \end{array} \right.$$

$$\text{Then } \frac{\pi d^2}{4} \times 10^5 = L \frac{\pi D}{p} \times 0{\cdot}7 \times 54{,}000 \times \lambda$$

$$\text{Whence we find that } D = \text{approx. } \frac{p}{1{\cdot}5\lambda} d$$

giving a rough figure with which we can start a design.

We also know that the depth of slot = 0·04D on the assumption that $\frac{\text{pole-face density}}{\text{tooth-root density}} = \frac{1}{2{\cdot}7}$; and that the width of top of tooth = width of slot.

In any case, assuming that there is a maximum tooth-root density, then, inserting that limit, we arrive at a depth of slot which is *entirely independent* of the number of slots used;—a point of great importance.

On the other hand, if a square pole be adopted, we have—

$$d^2 \times 10^5 = L \times \frac{\pi D}{p} \times 0{\cdot}7 \times 54{,}000 \times \lambda$$

$$D = \frac{p}{1{\cdot}2\lambda} L, \text{ or } \frac{p}{1{\cdot}2\lambda} d$$

if d be called the length of side of the pole.

It may also be pointed out that if the flux in any part of a machine is once determined, the whole proportions of the magnetic

* For meaning of symbols, see list at the beginning of the book.

circuit follow as a matter of course if the material used is known. Thus we may make a table of relative areas of the parts of the magnetic circuit.

TABLE I.

Part.	Flux density.	Relative area.
	Per square inch.	
Pole face . . .	50,000	2
Pole	100,000	1
Yoke—cast steel .	80,000	$\frac{5}{8}$
Yoke—cast iron .	30,000	$\frac{5}{3}$
Teeth . . .	120,000 to 150,000	([illegible] to $\frac{2}{3}$) $\div \lambda$
Armature below teeth	50,000 to 90,000	($\frac{5}{9}$ to 1) $\div \lambda$

Where λ is the Hopkinson leakage factor.

General Relationship between Output and Dimensions.—The output which can be obtained from a frame proportioned according to the rules expressed in Chaps. I. and II., depends chiefly upon two considerations, which as largely influencing also the relative proportions of the various parts of the machine, must be discussed before turning to the general question.

The two considerations referred to are, (1) Temperature rise, (2) Commutation or sparking limit.

Now the E.M.F. equation for a generator may be expressed as follows :—

$$\text{Volts} = \frac{\text{revolutions per minute}}{60} \times \frac{\text{poles}}{\text{armature circuits}} \times \frac{\text{total armature conductors} \times \text{flux per pole}}{10^8}$$

If both sides of this equation be multiplied by the total current we get—

$$\text{Watts} = (\text{poles} \times \text{flux per pole}) \times (\text{conductors} \times \text{current per conductor}) \times \frac{1}{10^8 \times 60}.$$

(Poles $\times$ flux per pole) is sometimes called the total magnetic armature loading, and is denoted by Y.*

(Conductors $\times$ current per conductor) is sometimes called the total electric loading of the armature, and is denoted by X.*

* Compare Macfarlane and Burge, *Journal I.E.E.*, vol. XLII., p. 238.

$$\text{Thus} \frac{\text{watts output}}{\text{R.P.M.}} = \text{specific torque} = \frac{XY}{60 \times 10^8}$$

Now poles $\times$ lines per pole $= \pi \times D \times \dfrac{\text{pole-arc}}{\text{pole-pitch}} \times$ axial-length of pole-face $\times$ density at pole-face; and—

armature-conductors $\times$ current per conductor
$= \pi D \times$ ampere-conductors per inch (of armature periphery).

Substituting these we get the relationship—

$D^2 \times$ axial length of pole-face

$$= \frac{60\cdot8 \times 10^7 \times \text{pole-pitch}}{\text{density at pole-face} \times \text{amp. condrs. per in.} \times \text{pole-arc}} \times \frac{\text{watts}}{\text{R.P.M.}}$$

If we now put in the average values for these quantities, which cannot change much, as for instance—

Density at pole-face, say, 54,000 lines per square inch.
Ratio pole-arc $\div$ pole pitch, say, 0·7.

$$\text{Then } D^2 \times L \times \text{ampere-conductors per inch} = 16000 \times \frac{\text{watts}}{\text{R.P.M.}}$$

If we adopt poles of circular section, it has been already shown, with the foregoing constants, and assuming the length of pole-face = d, that—

$$D = \frac{p}{1\cdot5\lambda} d, \quad \text{or } d = \frac{1\cdot5\lambda D}{p}$$

$$\text{Hence } \frac{D^3\lambda}{p} \times \text{ampere-conductors per inch} = \frac{10{,}700}{\lambda} \frac{\text{watts}}{\text{R.P.M.}}$$

$$\text{or for square poles} = \frac{13{,}370}{\lambda} \frac{\text{watts}}{\text{R.P.M.}}$$

The importance of the value of the ampere-conductors per inch (q) is here rendered very clear, but as the controlling factors—heating and commutation—both depend upon it, it cannot be rigidly fixed.

For small machines in modern practice it varies between very wide limits, but for large machines (over 500 watts per rev. per min.) it has an almost constant value. Indeed, it only varies between 700 and 950, so that if we insert a mean value of 800 in the formulæ we have for large machines the extremely useful relationship—

$$D^2L = 20 \frac{\text{watts}}{\text{R.P.M.}}$$

Of course the above constants are empirical to this extent, that they are deduced from values found in modern practice to give satisfactory results in the case of machines subject to limitations under the two headings "commutation" and "heating." They cannot, therefore, represent the best that can be done with interpoles, nor

should they be used in any case for other than obtaining preliminary dimensions or for checking final calculations.

An example of the use of the above will make these limitations clear. Suppose we wish to design a 200 K.W. 500 volt machine to run at 400 R.P.M.

$$\text{Then } D^2L = 20\frac{200{,}000}{400} = 10{,}000$$

Now, with circular poles—

$$D = \frac{p}{1{\cdot}5\lambda}d = \frac{p}{1{\cdot}5\lambda}L$$

$$\therefore \text{ for 4 poles, } L = \tfrac{3}{8}D\lambda$$

$$\text{6 poles, } L = \tfrac{1}{4}D\lambda$$

And the machine will probably have either 4 or 6 poles.

$$\begin{aligned}
\text{If 4 poles, } D^3\lambda &= 26{,}700 \\
D &= \text{nearly } 29'' \ (\lambda = 1{\cdot}15) \\
D^2L &= 10{,}000,\ L = 12'' \\
\text{If 6 poles, } D^3\lambda &= 40{,}000 \\
D &= \text{about } 33'' \\
L &= 9'' \text{ practically}
\end{aligned}$$

Now, it is evident that D^2L is a measure of the armature volume, and in these two cases the value is about the same. There will, however, be differences in the two machines in the following respects:—

(1) Peripheral speed.

(2) Division of losses, for in the six-pole machine the iron loss will be greater and the armature copper loss probably less.

(3) Surface available for radiation of heat slightly different.

(4) Total field ampere-turns, which will be greater in the six-pole machine, since the air-gap length and densities remain about the same.

(5) Ratio armature ampere-turns to field ampere-turns.

(6) Size of field-frame.

Therefore before we can even decide as to the number of poles or the shape of the machine all the above questions must be examined. We may add here that in the case of small machines not even an approximation like the above can be obtained. For neither the value of the ampere-turns per inch nor of the magnetic densities is even approximately constant, so that other methods must be adopted.

Now, all of the above differences, except Nos. (1) and (6), affect not only the *actual* but the *relative* proportions of the armature parts dealt with in the next chapter. The size of field frame is considered elsewhere, but the peripheral speed question may be dealt with here.

Armature Peripheral Speed.—Except in the case of Turbo-generators (which are not here considered) the armature peripheral speed has never been a serious limiting factor in dynamo design. On the contrary, economical machines usually have a peripheral speed which does not exceed 4000 feet per minute. Special precautions would have to be adopted if this figure were to exceed 5000 feet per minute, but there is no necessity for the use of such a high value.

Commutator Peripheral Speed.—On the other hand, with a peripheral speed of commutator exceeding 3500 feet per minute, chattering of the brushes and consequent difficulties with commutation are always likely to occur; and this figure does often come in as a very real limit to the machine dimensions.

CHAPTER III

RELATIVE PROPORTIONS OF THE ARMATURE PARTS

THE proportioning of the armature is affected by eight considerations:—

1. **Efficiency, and 2, division of losses for a given temperature rise.**
3. **Pressure and current.**
4. **Standardization.**
5. **Appearance.**
6. **Ampere-turns of the armature.**
7. **Commutation.**
8. **Use of Neutralization.**

1. Efficiency, and 2, division of Losses for a given Temperature Rise.—If the efficiency be decided upon, the proportion of watts lost in the armature to those lost in the field is determined by two considerations, viz.—

(I.) Efficiency characteristic required.

(II.) Whether the machine is to be sometimes totally enclosed.

(I.) **Efficiency Characteristic required.**—The losses of a machine may be practically subdivided into—

(*a*) **Constant Losses**, consisting of friction, hysteresis, and eddy-currents, and the shunt-field loss. These losses are not *actually* but are *practically* constant; and

(*b*) **Variable Losses**, consisting of armature and commutator C^2R loss, and series-field loss.

The efficiency of a machine is a maximum when $a = b$. Thus the division of losses determines the load at which the efficiency is a maximum. The choice of this load will depend upon the nature of the service required. Thus a dynamo with an average load = 75 per cent. of its full load, should have its maximum efficiency at this point, and so on. Figs. 18 and 19 show average values of the commercial efficiency of standard machines at full load; and Fig. 17 shows the variation due to speed.

At the point of maximum electrical efficiency, the preliminary division of losses for a shunt machine is determined by

$$*(1)\ R_1 = R\frac{1 - \eta_1^2}{4\eta_1}$$

$$*(2)\ Rf = R\frac{1 + \eta_1}{1 - \eta_1}$$

where R_1 = armature commutator and series field resistance,
Rf = shunt-field resistance corresponding to the load,
and η_1 = the electrical efficiency.

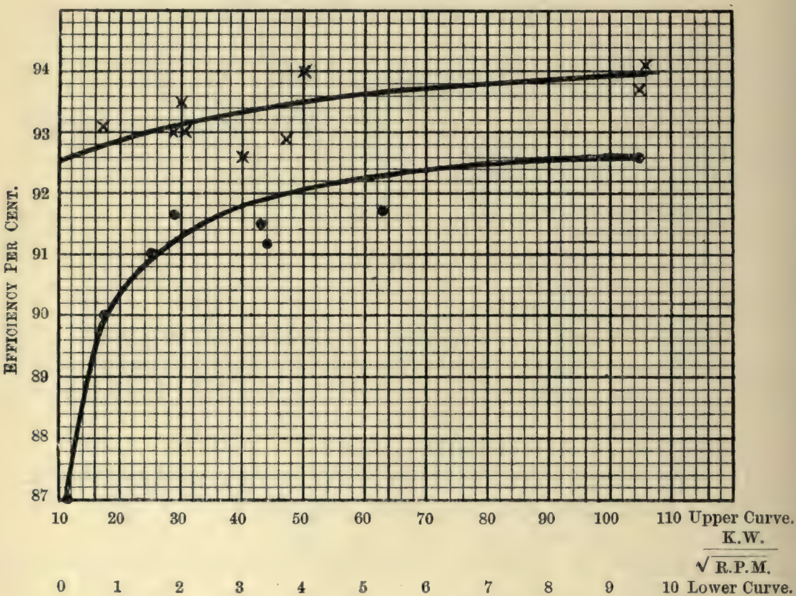


Fig. 17.—Curve between $\frac{K.W.}{\sqrt{R.P.M.}}$ and Efficiency.

Since, however, not only the shunt losses, but all the constant losses may be included in an expression like $\frac{E^2}{Rf}$, we may extend the use of the above formulæ as follows:—

Let R_1, as before, be the resistance of the various paths in series carrying current (armature, compound winding, and commutator),

* See Appendix II.

Let R_2 be such a resistance that $\frac{E^2}{R_2}$ = the sum of all the constant losses, where E is the approximately constant terminal potential difference.

Let R be, as before, the external resistance corresponding to the load at which the commercial efficiency (η) is to be a maximum.

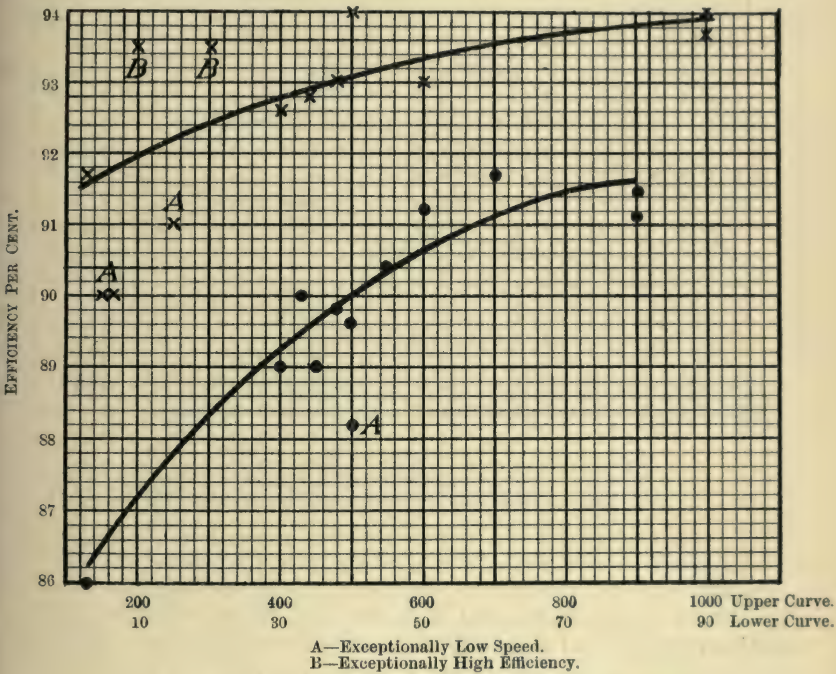


Fig. 18.—Curve between K.W. and Efficiency.

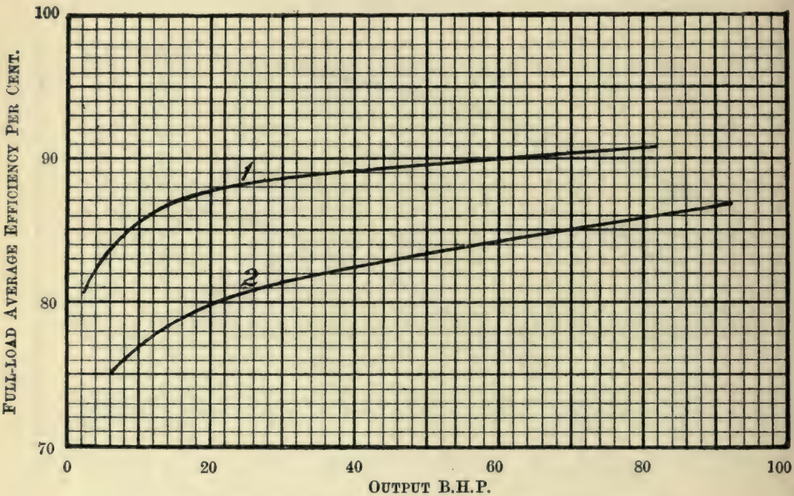
Then

$$R_1 = R\,\frac{1-\eta^2}{4\eta}$$

$$R_2 = R\,\frac{1+\eta}{1-\eta} \text{ for maximum efficiency.}$$

Now, of the various components into which $\frac{E^2}{R_2}$ is divisible, we can predetermine with considerable accuracy the commutator friction and the iron loss. The other friction losses vary usually from 0·5 per cent. in large machines to 2 per cent. of the output in small machines, and the balance of $\frac{E^2}{R_2}$ is the shunt-field loss.

These equations are only occasionally useful, since usually it is not the actual value R_2 that is needed, but only $\frac{E^2}{R_2}$. Consequently, more often the method illustrated in the following paragraph is of use.



(1) Full-load Efficiency Curves of Shunt and Compound Motors.
(2) Full-load Efficiencies of Series Motors, including Single Gearing.

FIG. 19.

Allotment of Constant Losses.—Suppose the commercial efficiency of a 200 K.W., 500 volt, 400 R.P.M. generator to have a maximum value of 93 per cent. at three-quarter full load.

Then constant losses and variable losses at $\frac{3}{4}$ full load = 7 per cent.
= 10·5 K.W.
And constant losses = variable losses = 5·25 K.W. at this load.

If we have an idea of the armature diameter and length (from such a preliminary equation, for instance, as that developed in the last chapter), then we also have an approximation to the size of the pole. From Table I. p. 21, we obtain the approximate section of the armature below the teeth, the depth of which may be roughly estimated. Then we have sufficient particulars for the iron losses, as will presently be seen; and from Chapter IX. p. 133, the commutator loss can be closely estimated. Thus we have to a first approximation not only the actual losses, but also the way in which they are to be allotted.

Applying the above as far as we are yet able, it has been shown that the armature diameter and length for a four-pole machine would be 29″ and 12″ respectively (p. 23). Taking depth of slot, say about 0·05D or 1·5″, we have—

TABLE II.

Diameter at tooth root	= 26″
Pole diameter	= 12″
Pole area	= 113 sq.″
Ratio of relative areas, say	= $\frac{5}{8}$
Armature area below teeth	= 70 sq.″ nearly (= $\frac{5}{8}$ × 113)
Armature nett length, say	= 10″ (= 12″ × 0·83)
∴ „ radial depth	= 7″
and internal diameter	= 12″
mean diameter	= 12 + 8·5 = 20·5″
Vol. armature, neglecting slots	= $\pi \times 20{\cdot}5 \times 8{\cdot}5 \times 10 = 5500$ cub. in.
Weight in lbs.	= 5500 × 0·28 = 1540
Frequency	= $\frac{2 \times 400}{60} = 13\frac{1}{3}$
Approx. density	= 60,000 lines per sq. in.

In a similar way the weight of the teeth can be calculated, and we need a method of determining the watts per lb. of iron in terms of the density and frequency. For "Stalloy" such curves have already been given. For ordinary armature iron we may obtain an exceedingly good approximation from a general formula as developed below. It is, however, again worth while calling attention to the fact that no approximations are very accurate for very small machines.

Estimation of Iron-loss.—It will be seen that in any case the predetermination of hysteresis and eddy losses is an important item in the design of continuous-current generators. Theoretical formulæ for this loss are very unsatisfactory and almost always lead to much too small a result. Careful experiments upon toothed core discs clamped together and running in a carefully measured field, yield the following formulæ, which can be used for estimating the loss:—

Iron-loss in watts per lb. = ~ per sec. × millions of lines per sq. in. × constant.

The constant here varies to some extent with the manner in which the volume of the armature is calculated in the test cases. If the tooth densities are not exceptional, and the armature is treated as a cylinder having an outside diameter = D and an inner diameter = the actual inner diameter of the discs, with a length = nett length of the stampings; if, moreover the density in this calculation is taken as the average value below the teeth, and the discs are not

more than 0·02″ thick;—then the constant varies in the author's experience between 1·7 and 1·9, and can easily be deduced from consistent tests on any given type of machine. With fair lamination 1·8 is a good value.

The theoretical formulæ are as follows:—

Hysteresis loss in watts per cubic inch = frequency $\times$ (density)$^{1.6}$ $\times 10^{-7} \times$ hysteretic constant

This hysteretic constant varies with the quality of the iron from about 0·0016 in first-rate material to 0·0032 in low-grade iron.

Eddy-current loss in watts per cubic inch = 150 $\times$ (frequency)^2 $\times$ (density)^2 $\times$ (thickness of plate)^2 $\times 10^{-12}$

The frequency in the above formulæ is in cycles per second, the density is in lines per square inch, and the thickness of plate in inches. Sometimes these formulæ will give a fair approximation to the loss, especially when the lamination is very carefully carried out, all burrs being avoided. They are especially useful for checking purposes, particularly when any of the densities used are abnormal; for the loss in any part (*e.g.* the teeth) can be separated from the rest of the losses.*

Of course, with any well-known brand of iron a series of curves like those in Fig. 16 can be used instead of the above formulæ.

Continuing the example on p. 29, and taking 1·9 as the constant in the iron loss formula, we get—

$$\text{Iron loss per lb.} = 13\tfrac{1}{3} \times 0{\cdot}06 \times 1{\cdot}9 = 1{\cdot}5 \text{ watts}$$
$$\text{Approximate loss} = 1{\cdot}5 \times 1540 = 2350 \text{ watts}$$

If the friction loss be 0·6 per cent., *i.e.* 1200 watts, we have 5250 − 3550 = 1700 for shunt-field and commutator friction losses.

The above illustration shows the effect of the losses on the efficiency curve. It would naturally be easier to design so that the full load efficiency is about a maximum, and this is more usually done, as will be illustrated in subsequent examples.

After subdivision of the constant losses as above, or in cases where such allotment is difficult, the curve Fig. 20 (whose shaded portion shows the ordinary limits of field losses in modern machines) may be used as a check or for approximate preliminary calculation.

Variable Losses.—The subdivision of the variable losses may be carried out as follows:—

The commutation resistance loss is at once determined by the commutator design (see pp. 133, 135). In the case of compound machines the armature and compound resistance losses are divided in any way most convenient. Usually the compounding is arranged to suit a standard shunt machine of definite output. Compounding

* See footnote, p. 204.

then reduces the shunt-field loss, increases the hysteresis loss (because higher saturations are reached), and transfers the balance of the constant losses as a variable loss to the field.

Series machines (usually traction, lift or crane, motors) are designed from an entirely different point of view, which will be considered later, but the values of their ordinary efficiencies have already been given in Fig. 19.

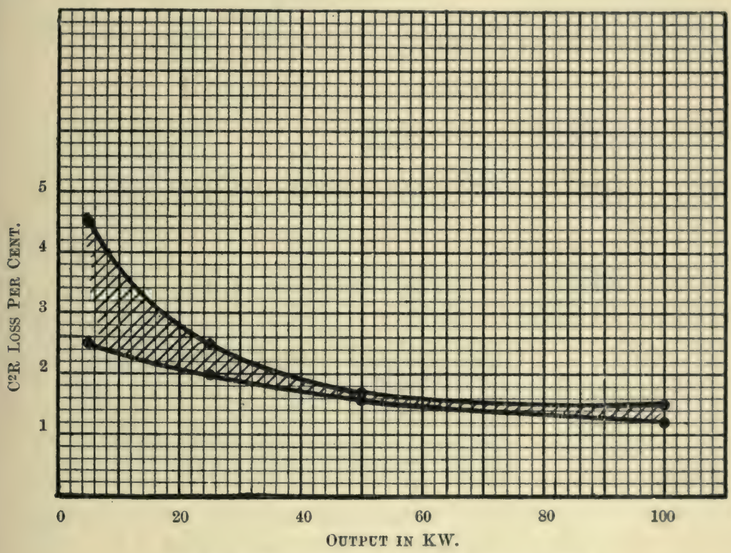


Fig. 20.—Curve of Shunt Field Loss.

(II.) **Effect of Enclosing.**—When a standard open-type machine is enclosed, the losses (and therefore the output) have to be lowered to keep the temperature-rise within reasonable limits. Reducing the output, however, practically only changes the variable losses; so that if these be small compared to the others, a large reduction of output only results in a small reduction of temperature-rise. Hence, if a machine is to give a good output per unit of weight when enclosed, the constant losses must be kept down. This is often the chief determining factor for the division of losses in small motors. The following may be taken as modern representative figures:—

The ratio variable losses : constant losses varies from 1 : 2 for a small 5 B.H.P. machine to 2·5 : 1 for a machine of 100 B.H.P. output.

The reason for this large variation is evident when it is recollected that whereas the smaller machine may require about 4000 ampere-turns on each pole of its field, the larger machine only requires

about 8000 ampere-turns, an increase not comparable with its increased output. Thus the constant field-loss is a much larger percentage of the total losses in the case of the smaller machine than it is in the case of the larger machine. On the other hand, machines of more than 30 B.H.P. rarely are required totally enclosed, and if they were, such sizes are worth while designing separately. So that this argument really affects small machines most, in which the above ratio varies at best from ½ at 5 B.H.P. to unity at 30 B.H.P.

Estimation of Copper Loss.—In *drum-wound armatures* the length of one conductor with its end-connection is nearly

$$\left(\text{armature core length} + 4{\cdot}5\frac{\text{D}}{p}\right)$$

The *armature copper loss* is then

$$(\textit{current per conductor})^2 \times \left(\textit{core length} + 4{\cdot}5\frac{\text{D}}{p}\right) \times \rho \times w \div \text{section of one conductor}$$

In this formula ρ = specific resistance of copper = 0·00000076 ohms per inch cube at 50° C.

In multiple-circuit armatures the current per conductor $= \frac{\text{C}}{p}$ (see Chap. VIII.).

In two-circuit armatures current per conductor $= \frac{\text{C}}{2}$ (see Chap. VIII.).

Combining the formulæ for copper and iron losses, we can find the total loss to be dissipated by the armature as heat. But the surface from which this heat will be dissipated is extremely difficult to estimate, and may be taken in a variety of ways.

The temperature-rise then depends upon the peripheral speed, the ventilation, and the number of watts per square inch to be dissipated. And this is a most important relationship to determine. The more so since, with the introduction of interpoles, the allowable temperature-rise becomes almost the sole factor limiting the size. A special chapter is therefore devoted to this question.

As a guide or check in the matter of armature (Fig. 21), C^2R loss is appended, which sets out, as does Fig. 20 for field losses, the usual range of armature-losses in practice.

3. Influence of Pressure and Current.—The design of a machine will vary greatly according as it is intended for high or for low pressure; comparing two machines of similar output, one of which is capable of giving 100 amperes at 400 volts, and the other 400 amperes at 100 volts, the commutator for the latter will require by far the larger surface, both for radiation and collection, to give the

same final temperature-rise. With regard to the armatures, more space will be taken up by the insulation in the case of the high voltage machine. Hence, the lower the current the shorter the commutator, and the higher the voltage the smaller the amount of space available for copper, so that an increase in the armature is necessitated in order to keep the same temperature rise. Thus it is convenient to lengthen the armature in the high voltage case. (It would be possible to increase the diameter of the armature but for the fact that a different size of disc for every different voltage would necessitate keeping a very large number of standard sizes of discs in

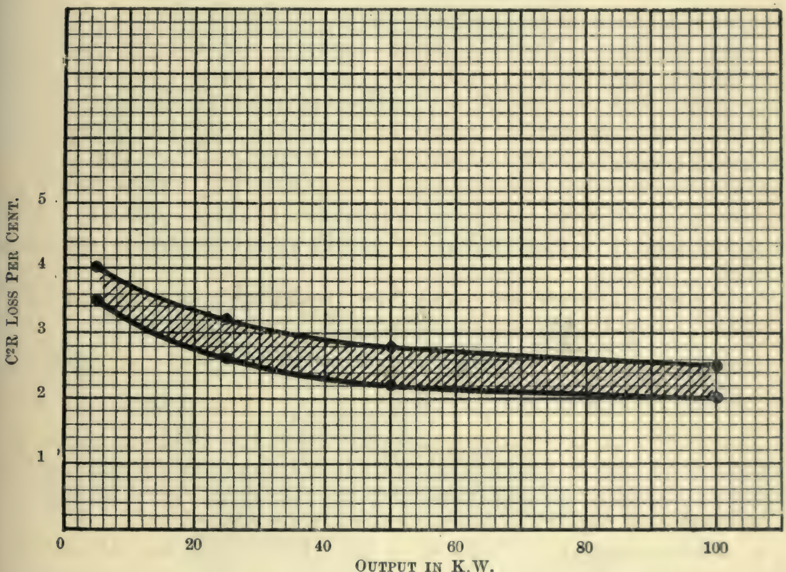


FIG. 21.—CURVE OF C^2R LOSS IN ARMATURE.

stock.) The amount by which the armature is lengthened is usually about that by which the commutator is shortened. In small machines this adjusting of the armature and commutator rarely pays. It is, indeed, better to keep the armature dimensions fixed, and adjust the output or the speed, or to leave room in the standard frame for the largest commutator that will be required. It will pay to shorten the commutator for the higher voltages, and the frame must be designed to take the commutator required by the lowest voltage.

4. Standardization.—The use of standard parts plays a very

important rôle in large works, and it is the duty of the designer to see that the number of standard sizes is kept as low as possible.

Whenever the output of a machine has to be changed by any considerable amount, it is best to get out a complete new series of designs, and find out therefrom the most economical proportions; having adopted these, slight changes in armature length will give necessary adjustment for current and voltage, etc.

5. Appearance.—The appearance of a machine is not always conducive to the most economical design, as we have already seen in the case of the Lahmeyer type referred to on p. 9.

The ratio of armature-diameter to armature-length being dependent upon the efficiency, upon the number of poles, and upon the cost, we not infrequently find that the most economical shape gives an ugly machine. Fig. 22 is an example of such a case. It represents a machine very economical as to material machinery and performance, but very difficult to sell on account of its appearance. It is practically impossible to make a machine of the same output in the round type so economical as the one illustrated, and yet such a design does not pay as compared with Fig. 23. This factor of appearance is always specially important in the case of small machines, and it is responsible for modifications in design referred to later.

6. Ampere-turns of the Armature.—These are usually calculated per pole, and evidently have the value—

$$\left.\begin{array}{c}\text{Ampere-turns of}\\ \text{the armature}\end{array}\right\} = \frac{\text{No. of conductors}}{2} \times \frac{\text{current per conductor}}{\text{poles}}$$

The values of the current per conductor for different types of winding are given in Chap. VIII., and it is impossible to discuss the armature-strength without reference to the field, so that further consideration of this point is reserved for Chap. V.

7. Commutation.—There is a limit to the self-induction of a coil undergoing commutation if sparklessness is to be maintained during the process. This limit is discussed under the heading of "Commutation," p. 128, where it is shown that the actual output of the machine can be put in terms of the turns per commutator section, the flux per pole, and the speed. Since these factors have a bearing upon the armature dimensions, it is obvious that from them a limiting expression can be derived as is done on pp. 131, 132.

8. Use of Neutralization.—If by means of "interpoles" or "commutating poles" (see pp. 54, 121) the armature reactions are reduced, the limit of output is changed, temperature-rise becomes the chief factor, and ventilation of the utmost importance. By such means the armature ampere-turns per pole may be greatly increased, or the field-strength per pole reduced; so that with the same field

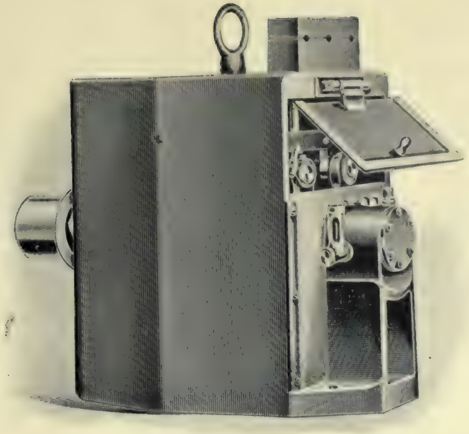


FIG. 22.—OBSOLETE BIPOLAR MACHINE (CRYPTO ELECTRICAL CO.).

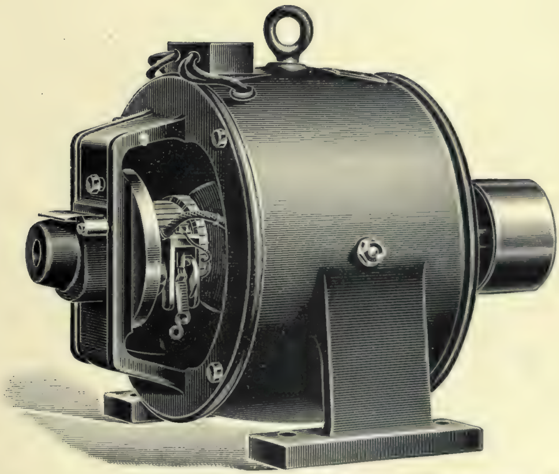


FIG. 23.—MODERN BIPOLAR MACHINE (CRYPTO ELECTRICAL CO.).

[*To face p.* 34.

loss much less copper may be used. It is found in practice that the cost of the material thus saved is considerably greater than the extra cost involved in fitting commutating poles, especially in large machines, so that commutating poles must henceforth form an essential part of large continuous current machines. For it must be borne in mind that such machines are limited in size, not only by temperature-rise but also by commutation, and, further, that the latter limit has been forced up by high air-gap and tooth densities. So that commutation affects the cost and dimensions more than is at first apparent; obviously in those instances where commutation was the limit, with commutating poles the output may be raised till temperature steps in, and then again it may be carried further by lower air-gap densities or shorter gaps, as is evidenced by Fig. 13. The saving obtained is well illustrated by the comparison carried out on pp. 189–196.

CHAPTER IV

RELATIVE PROPORTIONS OF THE FIELD MAGNET PARTS—FIELD CALCULATION

THE main proportions of the field-magnet are (as has been shown) dependent upon the materials used, upon economy in the use of those materials, and upon the flux per pole. There is, however, an exceedingly important dimension which depends upon other factors, viz. the length of the magnetizing bobbins or coils. Referring for a moment to Figs. 41 and 44, it will be seen that the coil length (l_c), depends upon a number of factors. For instance, it depends upon the position of the compound winding, if any (Fig. 41); and upon the use and shape of the spool or bobbin upon which the wire is wound. But more than either of these it depends upon the ampere-turns required for each coil, and upon the temperature rise of the coil that is allowed under normal working conditions. The latter factor is discussed in the chapter on temperature-rise; with the former we shall deal now.

On p. 14 a method for approximately determining the field ampere-turns per pole has been given. For final calculations of course a more accurate determination is necessary, and though the reader is assumed to be familiar with the principles of these calculations, we must work out a simple case here to render subsequent corrections quite clear.

Fig. 24 gives a dimensioned sketch of part of the field and armature of a multipolar machine. The path of the flux is from the pole F along the yoke C through the pole G, across the gap and teeth, and return by the armature-teeth and gap to the pole F. In parallel with the yoke C and armature D is a second path for the flux passing through the pole G, viz. *viâ* armature and yoke H and the next pole to the left (see Fig. 25). This, however, need not confuse the calculations, since it is clearly the business of the magnetizing coil around the pole G, to maintain such a magnetic potential-difference as will carry the flux through the required path from C, *viâ* pole G

and gap to the point D; and if it maintain this magnetic potential-difference on the one side, it will also maintain it on the other, *i.e.* towards H, since the two paths are in parallel. Thus each coil may be considered as responsible for such a part of the paths as that lying between the points C and D.

Let Ay be the sectional area of the yoke perpendicular to the lines of force.

Ap be the sectional area of the pole			do.	do.
Ag	,,	,, ,, gap	do.	do.
At	,,	,, ,, teeth	do.	do.
Aa	,,	,, ,, armature below the teeth.		

Let βy βp, etc., represent the magnetic densities in the sections

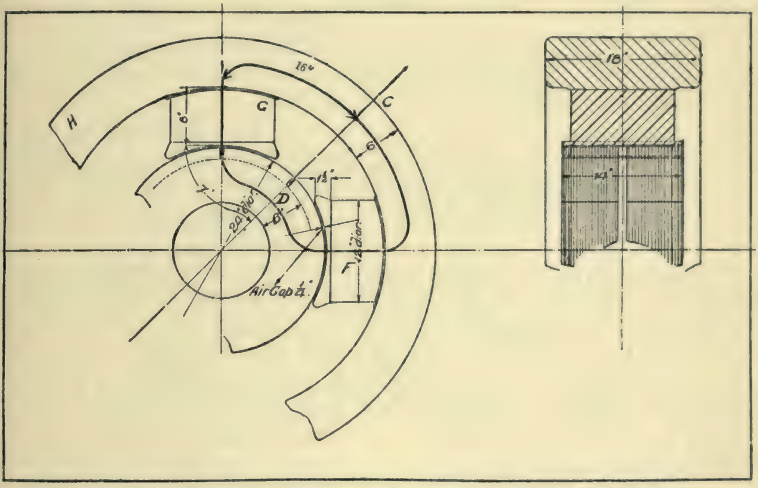


FIG. 24.—EXAMPLE OF A MAGNETIC CURRENT.

Ay, Ap, etc., and let the materials of which the circuit is composed be as follows:—

Yoke	cast iron
Poles	cast steel
Armature	lohys iron

The necessary dimensions will be found in Fig. 24, or in the details below:—

Slot depth, 1·3″	Number of slots, 96
Slot width, 0·3″	Yoke width parallel to shaft, 18″
Air-gap length, $\frac{1}{4}$″	Flux per pole = 10×10^6 lines

1. $Ay = 6'' \times 18'' = 108$ square inches.

Since the flux divides through the yoke, half going either way

$$\beta y = \frac{10 \times 10^6}{108 \times 2} = 43{,}100$$

From Fig. 6 the ampere-turns per inch corresponding to this density and this material are 100.

Therefore ampere-turns required for the yoke = 100 × 16 = 1600.

$$2.\ \mathrm{A}p = \pi \times \frac{12 \times 12}{4} = 113 \text{ sq. ins.}$$

$$\beta p = \frac{10 \times 10^6}{113} = 88{,}500$$

The ampere-turns per inch corresponding to this material and density are about 30 (Fig. 6).

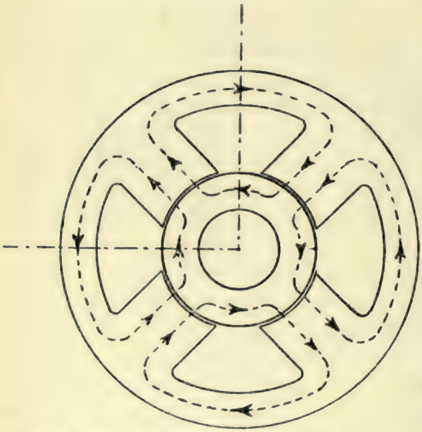


Fig. 25.—Magnetic-circuits in a Multipolar Machine.

Therefore ampere-turns required for the pole = 30 × 8″ = 240.

3. A*g*. The area of the gap may be, in the first instance, estimated as the mean area of the pole-face and of the tops of the teeth.

Now, area pole-face = 13″ × 14″ = 182 square inches.

Number of teeth under one pole-face = 16·8.

If 85 per cent. of the armature axial core-length be the nett length, *i.e.* be iron (the rest being ventilation spaces and insulation between laminæ), we have—

Axial length of one tooth = 0·85 × 14 = 12″ nearly

Width of tooth at armature-face = 0·485″

Area of flux-carrying teeth at armature-face = 16·8 × 0·485 × 12 = 97·5 sq. ins.

$$\text{Mean gap area} = \frac{97{\cdot}5 + 182}{2} = 139{\cdot}7 \text{ sq. ins.}$$

$$\beta g = 7200$$

$$\text{Ampere-turns for gap} = 0{\cdot}313 \times 7200 \times \tfrac{1}{4}'' = 5650$$

4. A*t*. This may be taken as the mean between the tooth area at the armature face and that at the tooth root. The section at the armature face has already been calculated as 97·5 sq. inches.

The width of the tooth at the root is = slot-pitch at tooth-root − width of slot.

$= (\pi \times \text{dia. at bottom of teeth} \div \text{No. of teeth}) - \text{width of slot}$

$= \frac{\pi \times (24 - 2{\cdot}6)}{96} - 0{\cdot}3 = 0{\cdot}4''$

$$\left.\begin{array}{r}\text{Area of teeth}\\ \text{at tooth-root}\end{array}\right\} = \text{area of teeth at armature face} \times \frac{\text{width at root}}{\text{width at face}}$$

$$= 97{\cdot}5 \times \frac{0{\cdot}4}{0{\cdot}485} = 80 \text{ sq. ins.}$$

$$At = \frac{97{\cdot}5 + 80}{2} = 88{\cdot}7 \text{ sq. ins.}$$

$$\beta t = \frac{10 \times 10^6}{88{\cdot}7} = 113{,}000$$

Ampere-turns for teeth per inch (from, say, Fig. 27) = 120

Ampere-turns for teeth = $120 \times 1{\cdot}3''$

= 156

5. Aa.

Area of the armature iron below teeth = Aa

= nett axial length of armature × radial depth

= 12×6

= 72 sq. ins.

Here again the flux per pole divides, half going either way.

$$\text{So } \beta a = \frac{10 \times 10^6}{72 \times 2} = 70{,}000.$$

Corresponding to ampere-turns per inch = 10

ampere-turns for armature = $10 \times 7''$

= 70

So total ampere-turns required to be provided by the field coil

= 1600 + 240 + 5650 + 156 + 70

= 7716 per pole

Note here, as nearly always, that the ampere-turns required for the armature iron are negligible.

Corrections to the above Calculation.—1. *Effective area of the air-gap.* This has been taken as the average of pole-face and tooth-tops, *i.e.* it is assumed that the flux is evenly distributed along the pole-face, stopping suddenly at the pole edges; and that it passes across the gap straight into the tops of the teeth only. Neither assumption is correct. From the edges of the pole-shoes a so-called "fringing" takes place, and the allowance for this fringing increases the area of the pole-face by an amount which by some writers is taken into consideration by adding one or two teeth to those actually

under the pole-face; others add 10 per cent. to the area calculated in the example. More correctly it may be considered by taking the pole-arc as—

Pole-arc + gap-length × constant

This constant depends chiefly upon the ratio—

$$\frac{\text{distance between pole-shoes}}{\text{length of air-gap}}$$

and values are given for it in the following table due to F. W. Carter:—*

TABLE III.

Distance between pole-shoes ÷ length of gap	4	5	6	7	8	9	10	12	14	16	18
Constant . .	1·32	1·59	1·79	1·98	2·15	2·3	2·43	2·65	2·84	3	3·15
Distance between pole-shoes ÷ length of gap	20	22	24	26	28	30	35	40	45	50	60
Constant . .	3·28	3·4	3·51	3·61	3·7	3·78	3·98	4·14	4·28	4·4	4·66

Example of Corrected Pole-arc.—Thus, according to this correction our air-gap area, not considering the slots, would be calculated as follows:—

$$\text{Distance between pole-tips} = \frac{\pi \times 24\cdot5}{4} - 13 = 7''$$

$$\text{ratio } \frac{\text{distance between pole-tips}}{\text{gap-length}} = \frac{7}{0\cdot25} = 28$$

$$\text{constant} = 3\cdot7$$

$$\text{effective pole-arc} = 13'' + 3\cdot7 \times 0\cdot25$$
$$= 13\cdot926''$$

$$\text{effective pole-arc area} = 13\cdot926 \times 14''$$
$$= 195\cdot59 \text{ instead of } 182 \text{ sq. ins.}$$

Effect of Slots.—F. W. Carter has also deduced a mathematical expression for the effect of the slots upon the area of the gap. According to his correction, the air-gap density is calculated upon the assumption of no slots existing, and this is then multiplied

* *Journ. Inst. Elec. Eng.*, vol. **29**, p. 436.

by a constant which is to be taken from the curves Fig. 26. It will be seen that the value of the constant depends upon the value of the slot-width, gap-length, and tooth-width.

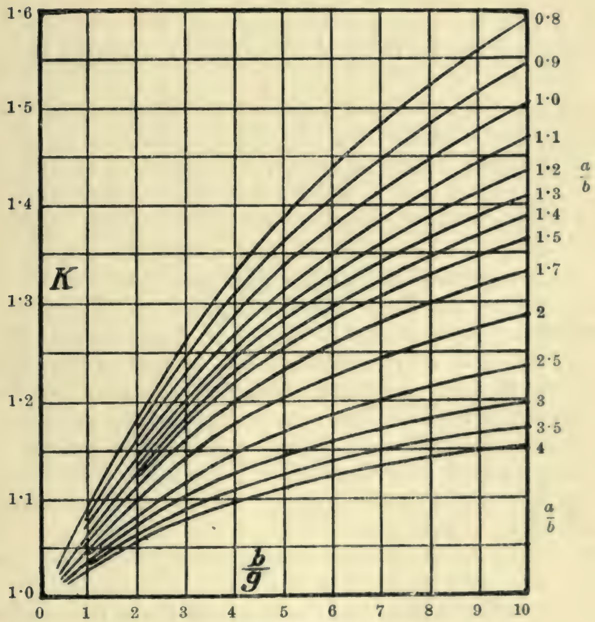
Applying this again to the case under discussion, we should have—

$$\text{Density in air-gap if there were no slots} = \frac{10 \times 10^6}{195} = 51{,}300$$

$$\text{ratio } \frac{\text{tooth-width}}{\text{slot-width}} = \left(\frac{\pi \times 24}{96} - 0{\cdot}3\right) \div 0{\cdot}3$$

$$= \frac{0{\cdot}485}{0{\cdot}3} = 1{\cdot}61$$

$$\text{ratio } \frac{\text{slot-width}}{\text{gap-length}} = \frac{0{\cdot}3}{0{\cdot}25} = 1{\cdot}2$$



a = Breadth of Tooth.
b = Breadth of Slot.
g = Length of Air-Gap.

FIG. 26.—EQUIVALENT AIR-GAP.

Constant from Fig. 26 = 1·075 approx.

Actual air-gap density = 51,300 × 1·075 = 52,500 (as against 72,000)

Corrected ampere-turns for the gap = 0·313 × 52,500 × 0·25
= 4140

As against the 5650 previously calculated, making a total of 6206,* instead of 7716 ampere-turns per pole. Thus the rough calculations are open to grave errors, and, though useful for preliminary trial designs, should always be checked by a closer calculation. In the same way (as has also been pointed out by Carter) the correction for ventilating spaces in the centre of an armature may be allowed for. Thus if in the present instance there were two half-inch air spaces in the armature core, the relationship of the width of these to half the remainder of the core would be similar to that existing between a slot and a tooth; we might say

$$\text{ratio} = \frac{13}{3} : 0{\cdot}5 = 8{\cdot}6 : 1$$

$$\text{Since also } \frac{\text{slot}}{\text{gap}} = \frac{0{\cdot}5}{0{\cdot}25} = 2$$

it is evident that the correction is negligible, and this is usually the case.

Correction for Tooth-density.—If all the lines do not pass into the tops of the teeth, then the mean tooth density as calculated in the first instance is also incorrect. In point of theory this is the case; but in point of fact it affects this total ampere-turns so little as to be negligible except in the case of very high tooth densities. For these latter, Hawkins, Hobart, and others have given empirical formulæ, but the present author has not found these to be reliable.

Ampere-turns for High Tooth-densities.—Often the densities in the teeth are so high that they cannot be read from ordinary curves like Fig. 6. In such cases recourse may be had to Fig. 27, which is the upper part of a curve for armature sheets drawn to a small scale.

II. Leakage Factor.—The above ampere-turn calculations again are only true on the assumption that no magnetic leakage exists, *i.e.* that the air acts as a perfect magnetic insulator. But if such were the case no flux could cross the gap, and further, perpetual motion would seem to be within reach.

Not only does no magnetic insulator exist, then, but none is even to be hoped for; and from this fact arises the necessity for making various allowances for flux-paths other than those just considered.

Wherever a magnetic potential difference exists, there a magnetic flux must also be established; and in air this flux will be directly proportional to the ampere-turns producing it, and to the area of the path, and inversely proportional to the length of the path.

* It is this increase in ampere-turns due to the teeth that leads designers to adopt half-closed slots like Fig. 86, 2, even when such a shape means putting the wires in one at a time. Verity's motors are sometimes so arranged, and require, consequently, a very small amount of field-copper.

Thus, if a number of ampere-turns act upon an air circuit, the flux set up is—

$$\text{Flux} = \text{ampere-turns} \times \frac{\text{mean area of circuit}}{0{\cdot}313 \times \text{mean length of magnetic path}}$$

In a circuit made up of iron and air in series, the iron part can often for a first approximation be neglected.

The mean area of a leakage-path and the mean length of magnetic line usually can only be roughly estimated. But the six general cases (given in Appendix III.) will be found convenient for reference and calculation.

Apart from calculations by such means as are illustrated in these cases, the most useful general formulæ are those evolved from the

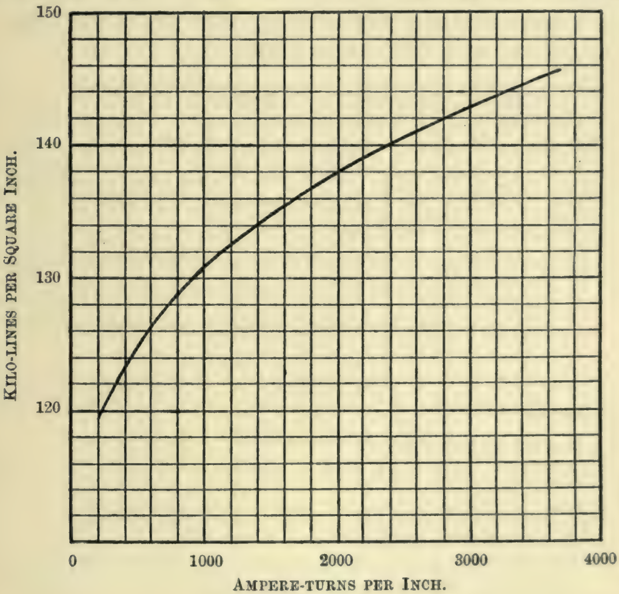


FIG. 27.

usual machine proportions by C. C. Hawkins.* The author prefers to express them here in a modified form as follows :—

Total leakage-flux per pole—

(*a*) For 4-pole machine = ampere-turns per pole $(7{\cdot}35d + D + 26)$

(*b*) For 6-pole machine = ampere - turns per pole $(8{\cdot}24d + {\cdot}7D + 26)$

(*c*) For 8-pole machine = ampere - turns per pole $(8{\cdot}37d + 5{\cdot}3D + 26)$

* Hawkins and Wallis, *The Dynamo*, 1903 Ed., p. 45.

where D and d are the armature-diameter and diameter of pole respectively in inches; or in the case of square poles, d = pole-side, as in Chapter I.

Value of Leakage Factor.—In Fig. 24 the chief leakage of flux would occur—

(*a*) From pole-tip to pole-tip.

(*b*) From pole to pole.

And since not only (*a*) but most of (*b*) takes place near the pole tip, it is not uncommon for designers to assume (following Hopkinson) that practically the whole leakage takes place from pole-tip to pole-tip; *i.e.* that the pole and yoke carry a flux λ times as great as that which passes across the gap and through the armature, where λ is called the (Hopkinson) leakage factor. The following are approximate values of this leakage factor for ordinary multipolar machines, and are useful for preliminary estimation purposes only:—

TABLE IV.—APPROXIMATE LEAKAGE FACTORS.

(*Machines of Medium Speed.*)

Kilowatts.	λ.	Kilowatts.	λ.
5	1·25 to 1·4	50	1.15 to 1·25
10	1·25 to 1·35	100	1·1 to 1·2
25	1·2 to 1·3	200 and over	1·08 to 1·15

Now, evidently—

$$\lambda = 1 + \frac{\text{number of leakage lines}}{\text{number of useful lines}}$$

$$= 1 + \frac{\text{reluctance of useful path}}{\text{reluctance of leakage path}}$$

$$= 1 + \frac{\text{ampere-turns for a given flux in the useful path}}{\text{ampere-turns to produce the same flux through leakage path}}$$

Now, the chief reluctances in the useful path are those of air-gap and teeth; hence

$$\text{Approximately } \lambda = 1 + \frac{\text{reluctance of air-gap and teeth}}{\text{reluctance of leakage path}}$$

where for any path of length l, and area = A, permeability = μ.

$$\text{Flux} = \frac{\text{magneto-motive force}}{\text{reluctance}} = \frac{\text{ampere-turns}}{0{\cdot}313 \text{ reluctance}}$$

Instead of reluctance we may write $\frac{1}{\text{permeance}}$; and this is often

more convenient, since the total leakage flux is generally the sum of leakage fluxes along several paths in parallel, as pole-tip to pole-tip, pole to pole, etc. It is the sum of these leakage fluxes we require, and since permeance is directly proportional to flux while reluctance is inversely proportional thereto, we can add the former directly for various parallel paths acted on by the same ampere-turns, while the latter cannot be so easily summed. This is, of course, quite analogous to the addition of "conductances" for parallel electric circuits.

Thus for calculation purposes the form

$$\lambda = 1 + \frac{\text{permeance of leakage paths}}{\text{permeance of air-gap and teeth}}$$

is most useful; where

$$\begin{aligned}\text{Flux} &= \text{magnetomotive force} \times \text{permeance}\\ &= \text{ampere-turns} \times \text{permeance} \div 0{\cdot}313\end{aligned}$$

Example of Approximate Leakage Factor Calculation.—We may calculate the approximate leakage factor for the frame shown in Fig. 24. The chief leakages will be from pole-shoes and from pole-cores, as already stated. The leakage from shoe to shoe divides itself into two parts, viz. that between the surfaces parallel to the shaft, and that between surfaces at right angles thereto. The first falls under Case I., p. 226. Thus—

Total ampere-turns acting across the gap between the shoes from one pole to the next = 2 ampere-turns per pole

$$= 12{,}412$$

$$\text{leakage flux} = 12{,}412 \times \tfrac{1}{2}\,\frac{14 \times 1{\cdot}5 + 14 \times 1{\cdot}5}{7 \times 0{\cdot}313}$$

$$= \frac{12{,}412 \times 21}{7 \times 0{\cdot}313} = 119{,}000$$

This takes place from each side.

$$\text{Total} = 119{,}000 \times 2 = 238{,}000$$

For the flanks of the shoes we may use Case III., p. 227.

The radius r will be half the polar arc, *i.e.* $= 6\tfrac{1}{2}''$.

$$\begin{aligned}\text{Thus flux} &= \frac{12{,}412}{0{\cdot}313}\,.\,2{\cdot}3\,.\,\frac{1{\cdot}5}{\pi}\log_{10}\left(\frac{\pi\,.\,6{\cdot}5 + 7}{7}\right)\\ &= 43{,}500 \times \log_{10} 3{\cdot}92\\ &= 26{,}000\end{aligned}$$

This again takes place from both flanks and in both directions.

The total leakage flux from this cause is then

$$= 4 \times 26{,}000 = 104{,}000$$

Leakage between pole and pole through the field-coils.

As the machine has circular poles, none of the cases given will

apply directly; we may, however, apply Case V., p. 228, if we allow for the fact that the poles are circular, by averaging the distance W. Thus average value of W (Fig. 28)—

$$= \mathrm{W} + 2\frac{4}{\pi}\int_0^{\frac{\pi}{4}} d \sin^2\theta \,.\, d\theta$$
$$= \mathrm{W} + 0{\cdot}36d = 20''$$

Also from Figs. 24 and 136; $h = 7''$; $l = 12$

$$\left.\begin{array}{l}\text{Leakage flux between}\\ \quad\text{pole and pole}\end{array}\right\} = \frac{12{,}412 \times 12}{0{\cdot}313 \times 7}\left(-\frac{7}{2} + \frac{2{\cdot}3 \times 20}{4}\log_{10}\frac{20}{20-14}\right)$$
$$= 68{,}000\,(-\,3{\cdot}5 + 11{\cdot}5\log_{10}3{\cdot}\dot{3})$$
$$= 68{,}000\,(2{\cdot}8) = 190{,}000$$

This takes place from the pole to each of its neighbours.
Total leakage flux per pole from this cause = 380,000.

Thus the total approximate leakage flux

$$= 238{,}000 + 104{,}000 + 380{,}000$$
$$= 722{,}000, \text{ or say roughly, } 8 \times 10^5 \text{ lines}$$

The flux per pole we have taken at 10×10^6 lines. If we regard this still as the value of the flux in pole and yoke, that in the air-gap and armature can only be $9{\cdot}2 \times 10^6$ lines, so that all the densities in the latter portions need to be recalculated. If, on the other hand, the flux in armature and air-gap be taken as 10×10^6, that in the pole and yoke must be $10{\cdot}8 \times 10^6$, and these densities must be recalculated. In either case the ampere-turns per pole are changed, which necessitates recalculation of leakage, so that only by successive approximations can the final values be approached. It is for this reason Table IV. is given, that by employing the value of λ taken from this table in the preliminary magnetic calculations and checking back, much subsequent correction may be avoided.

Assuming that 10×10^6 is the value of the flux in air-gap and armature, then that in the pole is—

$$10{\cdot}8 \times 10^6, \text{ and } \lambda = \frac{10{\cdot}8}{10} = 1{\cdot}08$$

The pole density corrected will be $88{,}500 \times 1{\cdot}08 = 95{,}500$; and the yoke density becomes 47,000.

These changes correspond to 2080 ampere-turns for the yoke and 340 ampere-turns for the pole, instead of 1600 and 240 respectively. Thus the total ampere-turns become—

2080 (yoke) + 340 (pole) + 4140 (gap) + 156 (teeth) + 70 (armature) = 6786 ampere-turns for ten million lines per pole in the armature, instead of 6206 by the previous calculation.

This alteration and recalculation could have been saved by the figure 1·1 from Table IV., after which the estimate of leakage would only have been carried out as a check.

It is to be noted here that any factor necessitating an increase in the ampere-turns per pole influences λ.

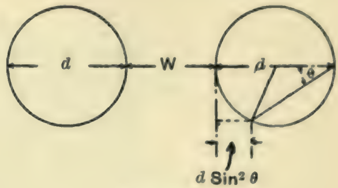


FIG. 28.

The ampere-turns just calculated are those corresponding to no-load. At full load λ is, of course, greater; but the estimation of full-load ampere-turns belongs to a later chapter; here it is of interest to compare the calculation just carried out with the approximation given by the formula of Hawkins.

$$\begin{aligned} \text{Leakage flux} &= 6986(7{\cdot}35 \times 12 + 24 + 26) \\ &= 96 \times 10^4 \\ \text{leakage factor} &= 1{\cdot}096 \end{aligned}$$

which compares well with our previous value.

Leakage Flux at the Mouth of a Slot (F. W. Carter).—It is sometimes of importance (as in the case of interpole machines) to calculate the flux set up at the mouth of a slot which is itself under a pole-shoe. In such a case the flux across the slot through the ampere-wires *t. a.* (Fig. 29) can be calculated under Case IV. (p. 227),

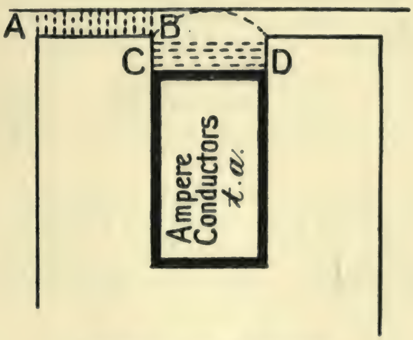


FIG. 29.—SLOT LEAKAGE.

and the flux which traverses the air-gap directly from the top of the tooth as at AB (Fig. 29) can be calculated, as also that which passes directly across the mouth of the slot at CD. Then that which springs from a corner such as B is obtained by multiplying the

ampere-wires of the slot by the appropriate constant taken from the curve, Fig. 30, and by the net length of the armature (or interpole) in inches.

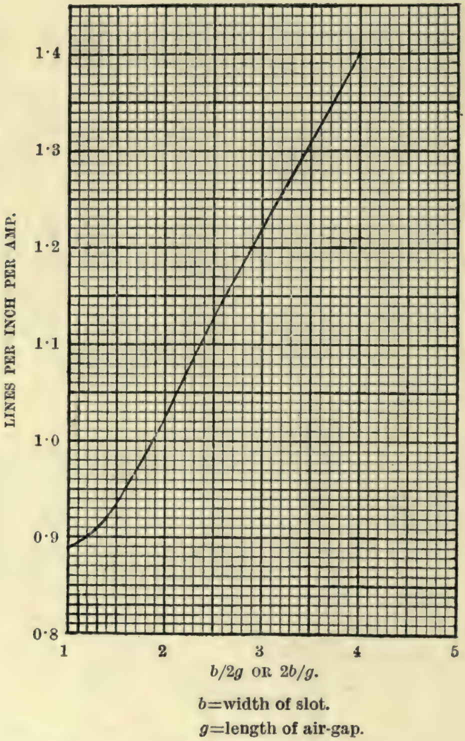


FIG. 30.—SLOT LEAKAGE—LINES UNDER POLE-FACE.

Magnetization Curve.—Just as the ampere-turns corresponding to ten million lines per pole have been found to be 6986 for the machine in Fig. 24, so for any other flux per pole can the corresponding ampere-turns be found. These can then be plotted in the form of a curve as shown in Fig. 31. Such a curve is known as a "magnetization characteristic." It is evident (since at constant speed with no load and a constant number of armature conductors the terminal volts are proportional to the flux) that for a certain armature winding and speed the ordinates of the curve may be changed to a volt scale, and when the machine is built such a curve may easily be obtained on test and compared with that predicted. Not only for this reason, but for its use in calculating

the compounding ampere-turns, such a curve should always be drawn out for a new machine.

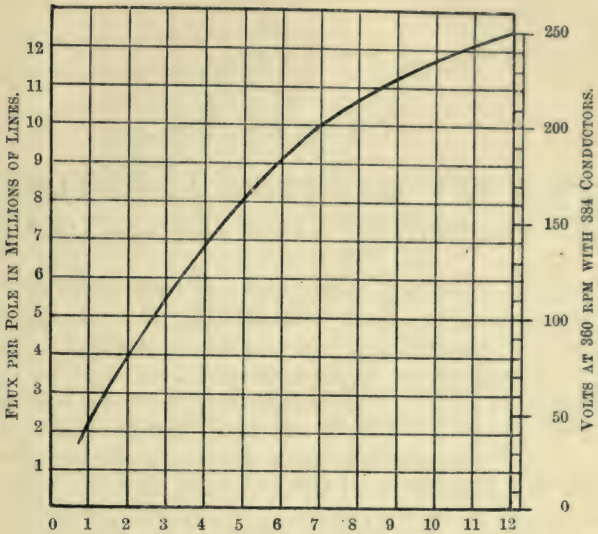


FIG. 31.—MAGNETIZATION CURVE.

Interpole Calculation.—The calculations involved in connection with interpoles fall more appropriately under the heading of commutation than of field-magnets. They are consequently more fully dealt with in Chap. IX., and in the examples of procedure in design, especially pp. 194–197 and p. 209. The former pages show the effect on a standard design which interpoles produce.

CHAPTER V

RELATIONSHIP BETWEEN ARMATURE AND FIELD STRENGTH—FIELD CALCULATION

FROM the calculations and corrections just considered, and from the temperature rise factors dealt with in Chap. VI., the dimensions of the field for a given flux per pole can be determined. The dimensions of the armature for a given output have also been outlined, and we proceed to consider the relationship between field and armature. Generally speaking there are two ways in which the dimensions of one of these parts influences the other. It has already been shown that the dimensions of the armature iron as forming part of the magnetic circuit are dependent upon the flux per pole on account of the phenomenon of saturation; but the point which has not been approached is the relationship between field "strength" and armature "strength"; that is, between the ampere-turns of the field per pole, and the corresponding ampere-turns of the armature. The total of the ampere-conductors of the armature is called (as already stated) the "electric loading," and the relationship therefore which we refer to is closely connected with the ratio $\frac{\text{magnetic loading}}{\text{electric loading}}$. Some short elementary introduction is necessary to make subsequent reasoning clear.

Armature Reaction.—So far our calculations have proceeded without any reference to the actual armature winding, and the ampere-turns have been calculated for the field without considering the "back" ampere-turns produced by the armature.

Now, for a given output, flux per pole, and speed, the armature ampere-turns are fixed whatever the values of the current and voltage may be. Thus if we double the voltage we halve the current; but the number of conductors must be increased, with the result that the number of armature ampere-turns remains practically the same.

These armature ampere-turns cannot exist without tending to set up a magnetic field, for they convert the armature into a magnetic solenoid whose axis is along a line passing through the conductors under commutation. This line we call the *Brush Axis.* A glance

at Fig. 32 (which shows the current-circulation and armature field for a bi-polar machine) explains the "armature reaction." All the wires on the one side of the brush axis carry current in one direction,

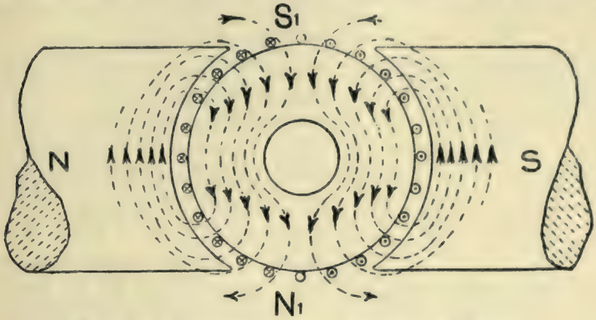


FIG. 32.—DISTRIBUTION OF ARMATURE FIELD.

whilst those on the other side carry current in the opposite direction, resulting in the magnetization of the armature along a line determined by the brush position.

The flux due to the armature ampere-turns is obviously roughly at right angles to the field flux.

Suppose provisionally the reluctance of the magnetic circuit to be the same all round the armature. Let this reluctance be R.

$$\text{Then total flux of the field magnets} = \frac{\text{field ampere-turns}}{R}$$

$$\text{Flux set up by the armature} = \frac{\text{armature AT}}{R}$$

The direction of the resultant field due to the two magnetizing forces may be obtained in direction by drawing a simple vector diagram with AT_f to represent the field ampere-turns, and AT_a to represent the armature ampere-turns (Figs. 33 and 34). If the reluctance is constant, this resultant gives the total acting ampere-turns. In practice AT_a is not on the same scale as AT_f, because of the different reluctances of the paths along which these ampere-turns act.

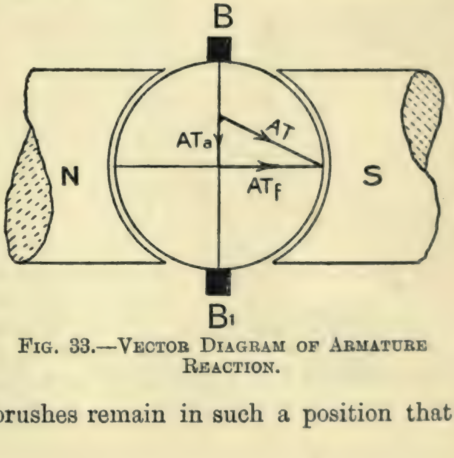


FIG. 33.—VECTOR DIAGRAM OF ARMATURE REACTION.

So long, however, as the brushes remain in such a position that

the armature magnetic axis is at right angles to the field magnetic axis, little or no weakening of the main field results. But, if they be moved ever so little one way or the other, there is immediately a component of the armature ampere-turns tending to weaken or strengthen the main field. Usually, commutation demands a move in such a direction as to weaken the field, called "forward lead," movement in a reverse direction being called "backward lead."

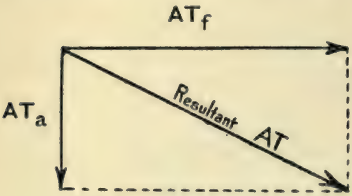


FIG. 34.

When the brushes are in such a position that the axis of the conductors under commutation is also the axis of the *resultant* field, they are said to be in the "neutral position," *i.e.* the line joining the brushes in Fig. 33 should also be at right angles to the line marked AT. To obtain this the brushes must obviously be moved in the direction of rotation.

With the brushes in the positions BB^1 (Fig. 33), and taking the reluctance as constant, then—

$$\text{Resultant AT} = \sqrt{AT_f^2 + AT_a^2}$$

The resultant is thus increased in magnitude, but altered in direction as AT_a increases. Although the resultant field is thus greater than that due to AT_f, the E.M.F. induced in the armature will not be larger because it is only the component of the field at right angles to the brushes which can produce any E.M.F. *at* the brushes; the other component, being in a line with the brushes, has no effect on the E.M.F. between the brushes.

Field Diagram with Brushes moved forward to the Neutral Position.—If we move the brushes forward until they are in the new "neutral position," *i.e.* until they are at right angles to the resultant field just obtained, then the direction of the armature field will also be changed, being moved in the same direction as the brushes. The resultant field will now be changed again, being moved round a little further.

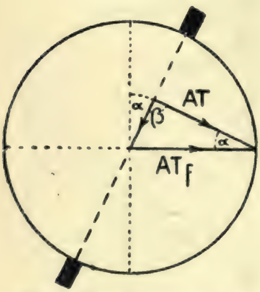


FIG. 35.—BRUSHES IN TRUE NEUTRAL POSITION.

The brushes in practice are often moved forward until they are in the actual neutral position (Fig. 35).

To show that the Brushes may be obtained in the Actual Neutral Position.—From the above it appears that moving the brushes round will move the resultant field round also, *i.e.* that the brushes can

never be got into the neutral position. This, however, is not the case.

Imagine the brushes moved forward to some position in advance of that shown in Fig. 35. The angle β would then be greater than a right angle, so the brushes would be in advance of the neutral position, for the two vectors AT_a and AT_f are supposed constant in magnitude.

Resultant Field.—With the brushes moved forward to the neutral position it is seen from Fig. 35 that the resultant field AT is less than AT_f, besides being moved round through an angle a.

In Fig. 35 if the brushes be in the neutral position, *i.e.* at right angles to the resultant field, then—

$$AT_f^2 = AT_a^2 + AT^2$$
$$\text{or } AT^2 = AT_f^2 - AT^2$$

Also let a = the angle through which the brushes are moved forward to the neutral position, then—

$$a = \sin^{-1}\frac{AT_a}{AT_f}$$

Thus if we know AT_a and AT_f, a may be found. Also knowing a and AT_f, AT may be found.

This solution by vectors, although it gives a good idea of what is happening is not of much use in practice, because, as has been said, the reluctances of the two flux-paths are not the same, *i.e.* we cannot write AT_a and AT_f to represent the fields unless they are measured on different scales.

In order to take into account the very different reluctances along the field axis and at right angles thereto, it is better to consider the

FIG. 36.—CROSS AND BACK AMPERE-TURNS.

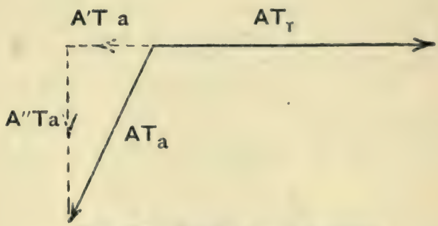


FIG. 37.—RESOLVED ARMATURE REACTION.

armature ampere-turns as divided up in accordance with Figs. 36 and 37.

The total conductors do not now act as one complete "solenoid," half the conductors being on each side, but they are split up into

two "solenoids," as indicated by the dotted and chain-dotted lines. The small "solenoid" composed of the conductors at the top and bottom produces a field *in a line with* the main field but opposite in direction AT'_a. The other "solenoid" produces lines at right angles to the main field. Thus these turns are known as the back ampere-turns and the cross ampere-turns respectively. From this it is easily seen that the more the brushes are moved forward, the greater will AT'_a be, so that the resulting field will be weaker.

Let T_a = total armature turns

and a = angle through which the brushes are moved (in degrees).

Then total No. of conductors $= 2T_a$

$$\text{Total back turns} = \frac{2T_a\,(2a)}{360}$$

$$= \frac{4aT_a}{360}$$

For a bipolar machine the current per conductor = $\frac{1}{2}$ total current

$$\therefore \text{ Back amp. turns} = \text{current per conductor} \times \frac{4aT_a}{360}$$

If C = total current

$$\text{Back amp. turns} = \frac{C}{2} \times \frac{4aT_a}{360} = \frac{2aT_a \times C}{360}$$

$$\text{Back amp. turns per pole} = \frac{aT_a \,.\, C}{360}$$

Also cross ampere-turns = total armature ampere-turns − back ampere-turns (arithmetical difference).

Back AT in Multipolar Machines.—With the brushes moved forward through an angle = a (Fig. 38), the current in the conductors has the direction indicated. The back AT then consist of the conductors lying under angle = $2a$, as indicated by the dotted connecting lines.

The back ampere-turns per pole are from this diagram seen to be in general—

Current per conductor $\times \frac{a}{360} \times$ total armature conductors, where a is

the *actual* angle of brush lead.

Use of Neutralization.—It is evident from Fig. 32 that if a solenoid of equal strength to the armature be caused to act in opposition thereto along the brush axis, nearly all the effects of armature reaction may be eliminated and the foregoing calculations much simplified. This neutralization is accomplished in various ways which, however, are considered under the heading of commutation (Chap. IX.,

Figs. 78 and 79). The most ordinary method is by means of so-called interpoles (p. 121), and where these are properly arranged no addition to the main field ampere-turns, calculated as in Chap. IV., need be made. Where these are not used the total ampere-turns on the main field coils must allow for the armature reaction. There is

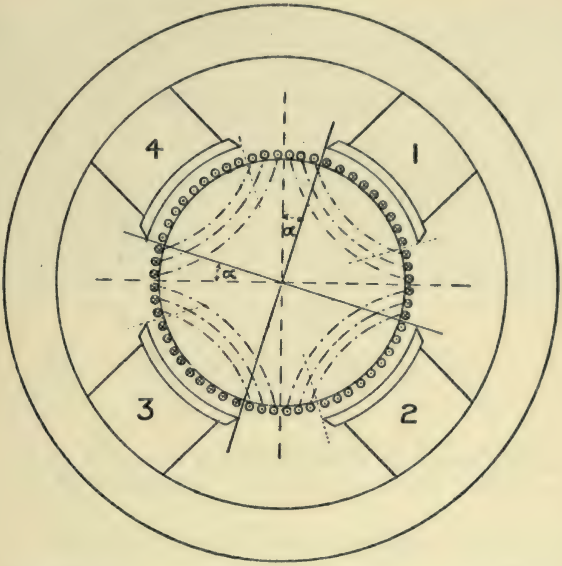


FIG. 38.—ARMATURE REACTION IN MULTIPOLAR MACHINE.

no very satisfactory means of doing this, but those methods most commonly in use will be found below.

To calculate the Total AT for the Field.—There are in practice two methods of calculating the addition to the ampere-turns required to compensate for the armature reaction.

(1) Assume the brush lead to be so great that $2a$ = angle between pole tips. AT_b is then calculated from this assumption, the cross AT being neglected.*

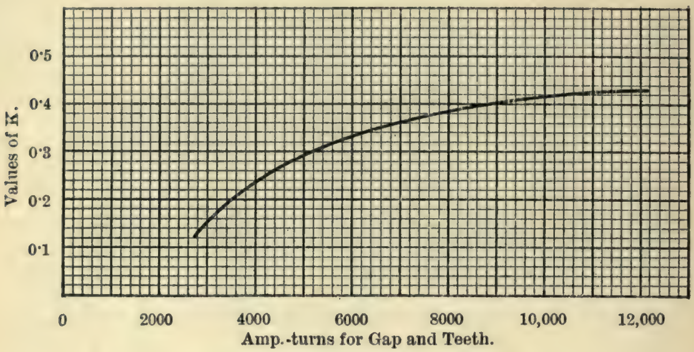
$$\text{Total AT} = AT_f + AT_b$$

This method allows ample AT, for the brushes are really never so far forward.

* It should be obvious, from Figs. 36 and 37, that the cross ampere-turns cannot reduce the main field strength *directly*, but only *indirectly*, by altering the saturation in the teeth or pole-shoes. Thus where these saturations are not great the cross ampere-turns can safely be neglected. For a full detailed explanation of this point see S. P. Thompson's "Dynamo-Electric Machinery," vol. i. pp. 526, 527, 1904 edition.

(2) (*a*) The actual brush lead is known from a similar machine. The actual AT_b may be calculated and added to AT_f, and:—

(*b*) The cross AT are allowed for by reference to experimental curves, such, for instance, as Fig. 39, which purports to give the number of ampere-turns (called "compensating" ampere-turns) to be



$$K = \frac{\text{compensating amp.-turns}}{\text{distorting amp.-turns}}$$

FIG. 39.

added to the field-coil to counterbalance the effect of the armature cross ampere-turns as increasing the saturation of the magnetic circuit.

Examples of the application of these corrections are given on pp. 59 and 219.

Field Coil Calculations.—The calculations for the ampere-turns corresponding to a given flux per pole have been explained and illustrated in Chap. IV., and those required for armature reaction have just been estimated. It is necessary here to point out that the flux per pole must be large enough to provide for the maximum E.M.F. that the machine will ever be called upon to produce. Thus, losses of pressure due to load must be allowed for; such as those occasioned by the resistance of the armature windings and of the commutator (CR_a and CR_c), as also any loss due to fall in speed as load comes on, etc. In a machine possessing only shunt coils all the ampere-turns required by these various items must be provided by the shunt winding, and regulation for the changing conditions must be obtained by a variable resistance inserted in series with these shunt windings. In a compound wound machine, on the other hand, the shunt winding is called upon to provide only the ampere-turns required to produce that flux which will give at normal speed and no load the proper open circuit volts.

The AT for the series coil consist (1) of the extra AT required to

increase the flux, so that the total voltage generated by the armature is = the no-load voltage + the CR drop in the armature and commutator, and in most cases also the CR drop in the series coils. Series turns also take into account—

(2) AT necessary to counteract armature reaction;

(3) AT required to increase the flux so as to make up for drop of volts due to slight fall in speed;

(4) Any rise of voltage required to allow for drop in feeders as the load comes on.

Of these AT those required for armature reaction are usually the greatest.

Having obtained the total AT required for shunt and series coils respectively, the watts to be expended in each coil are calculated, as suggested on pp. 30 and 186.

Shunt Coil Calculations.—Assume a constant P.D. across the field-coils. Then

$$\text{Field current} = C_f = \frac{E}{R_f}$$

where R_f = total resistance of all the coils.

If there are p poles, then

$$C_f = \frac{E}{p \times r_f}$$

where r_f = resistance per coil.

If r_{mt} = resistance per mean turn, and t = number of turns per coil, then

$$r_f = r_{mt} \times t$$

$$i.e.\ C_f = \frac{E}{p \times r_{mt} \times t}$$

$$\text{or } r_{mt} = \frac{E}{p \times tC_f} \quad . \quad . \quad . \quad . \quad (1)$$

and evidently the ampere-turns per coil required = tC_f.

Also—

$$\text{Ohms per yard of wire for field-coil} = \frac{r_{mt} \times 36}{(l_{mt})''} \quad . \quad (2)$$

where l_{mt} = length of mean turn in inches.

Allowances for Insulation on the Wire.—For small machines not above 200 volts, single cotton-covered wire provides sufficient insulation.

For single covering, 0·01″ must be added to the diameter of the wire. This allows not only for the space occupied by the covering, but also for the "spring" of the wire.

For large machines, and in any case above 200 volts, double cotton covering is required.

For D.C.C. add 0·018″ to diameter of wire.* Then

$$\text{Turns per layer} = \frac{\text{length of winding space}}{\text{diameter of covered wire}} \quad . \quad (3)$$

$$\text{And number of layers} = \frac{\text{depth of coil}}{\text{diameter of covered wire}} \quad . \quad (4)$$

$$\textit{turns per coil} = (3) \times (4)$$

When worked out, the calculations should be checked back by the equation—

$$r_f = \frac{\text{turns per coil} \times l_{mt} \times \text{ohms per yard}}{36}$$

$$\text{and } \mathrm{R}_f = r_f \times p$$

$$\text{and } \frac{(\mathrm{E})^2}{r_f \times p} = \text{watts lost in shunt field}$$

which should agree with the stipulated value.

Arrangement of Series and Shunt Coils.—By putting the series coil at the end of the pole, high saturation of the pole tips and therefore good regulation and commutation are obtained. However, the series coil requires a firm support, and when placed on the pole-end a very firm pole is required.

Typical arrangements are shown in Figs. 41, 42, and 43.

Construction of Compound Coils.—The series coils are usually made up of flat strips, generally wound on edge. Round multiple wire cable is sometimes used, but the space factor then comes out lower than with strip.

In many series coils the size of the strip is so large that it is difficult to bend it properly, and then the series turns may be put all or partly in parallel.

Whenever the strips can be bent they should not be put in parallel, for if one joint is worse than the others, then there is an unequal division of the parallel currents, with consequent unbalanced fields.

Example of Preliminary Calculation of a Compound Winding.—Let us suppose that the machine in Fig. 24 is to be provided with a compound field winding; that the flux per pole in the armature at no load is 10×10^6 lines, and that the no-load voltage is 200. We have already shown (Fig. 31) that the ampere-turns necessary for this flux will be practically 7000 per pole, and it is wise even in a compound-wound machine to make provision for emergencies by having some resistance in the shunt field. Suppose that such a resistance absorbs 10 volts; then the pressure across the 4 shunt coils is 190 volts. Now, if the winding be arranged as in Fig. 43, p. 73,

* For more accurate details of insulation thickness, see p. 139.

we may take the depth of shunt and series winding as $2\frac{1}{4}$ inches about, and their lengths as two-thirds and one-third of the total winding length; *i.e.* (allowing for some insulation at the ends of the coils) as 5 inches and $2\frac{1}{2}$ inches respectively. The internal diameter of the winding (allowing for insulation between coil and pole) may be taken at $12\frac{1}{2}$ inches, so that the length of mean turn is $\pi \times (12\frac{1}{2} + 2\frac{1}{4}'') = 46{\cdot}4$ inches.

From the formulæ on p. 57, we have—

$$\text{Resistance of mean turn} = \frac{190}{4 \times 7000} = 0{\cdot}0068 \text{ ohm}$$

$$\text{ohms per yard} = \frac{0{\cdot}0068 \times 36}{46{\cdot}4}$$

$$= 0{\cdot}00528$$

This will be the resistance hot; the value cold may be taken as $0{\cdot}00528 \div 1{\cdot}16 = 0{\cdot}0046$ ohm.

The nearest wire to this is No. 14 S.W.G.

$$\text{Diameter bare} = 0{\cdot}08, \text{ covered } 0{\cdot}098$$

$$\text{turns per layer} = \frac{5}{0{\cdot}098} = 51$$

$$\text{layers} = \frac{2{\cdot}25}{0{\cdot}098} = 23$$

$$\text{turns per coil} = 1170$$

$$\text{total turns} = 4680$$

$$\text{yards} = 6000$$

$$\text{resistance, cold} = 28{\cdot}7 \text{ ohms}$$

$$\text{,, hot} = 33{\cdot}6 \text{ ohms}$$

$$C_f = \frac{190}{33{\cdot}6} = 5{\cdot}7 \text{ amperes nearly}$$

Watts to be dissipated by each shunt coil = 270.

Compound Winding.—We will first suppose that the various extras enumerated under the headings (1), (3), and (4), p. 57, amount to 20 volts. From Fig. 31 it is seen that for 220 volts the flux must be 11 million lines per pole, and that this corresponds to 9000 ampere-turns per pole. Thus the compound winding must provide for 9000 − 7000 = 2000 ampere-turns.

Back Ampere-turns.—Estimating these by method (1), p. 55, we have, if the output of the machine is 96 K.W., 120 amperes per bar, with a 4-circuit armature (Chap. VIII.). Then

$$\text{Back ampere-turns per pole} = \frac{384 \times 120 \times 0{\cdot}3}{2 \times 4} = 1728$$

Compound Winding.—So that total compounding ampere-turns per pole = 3728.

Now, the total current of the machine will be 480 amperes, and if this pass through the series turns, the turns per pole will be 8.

If we allow $1\frac{1}{4}$ volt drop in these series turns ($\frac{5}{8}$ per cent.),

$$\text{The resistance per turn will be } \frac{1{\cdot}25}{480 \times 8 \times 4} \text{ ohms}$$

and the section of the material will be at 50° C.

$$\frac{\text{specific resistance} \times \text{length mean turn}}{\text{resistance per turn}}$$

$$= \frac{76}{10^8} \times \frac{46{\cdot}4 \times 480 \times 8 \times 4}{1{\cdot}25} = 0{\cdot}435 \text{ sq.}''$$

This is heavy material to wind, but it could be done in a number of ways. Thus, a copper ribbon wound as illustrated in Fig. 43 could be adopted. Such a ribbon would need to be covered with braid or tape, and ought to be 2 inches wide by 0·218 inch thick; or better two ribbons in parallel each 2 inches wide by 0·0109 inch thick could be used; either of which arrangements would easily go into the space allotted. These calculations, while illustrating well the connection between dimensions of coil, ampere-turns, and watts wasted, do not take into account the question of temperature-rise. Further, nothing has been said as to whether the number of watts to be wasted by the shunt coil is right from the standpoint of efficiency. Consequently, the method given is chiefly of use in calculating the windings of standard machines in which the relationship of these quantities is known to be about right. The wider question is treated in Chap. VI.

The Limit of Output imposed by Armature Reaction.—It has been shown that when the armature is loaded, the current circulation is always such as to tend to form poles at points intermediate between the field poles, and this tendency results in the distortion of the main magnetic field. Forward lead of the brushes tends to accentuate this, and backward lead to diminish it. Even with no lead some distortion exists, unless the armature ampere-turns are compensated by one of the special devices referred to on p. 121. Where no such device exists, this interference largely determines the ratio of field ampere-turns per pole to armature ampere-turns per pole, and ultimately also the main dimensions of the machine. For it has been shown that, given the densities, the proportions are fixed; and the field magnet is usually more costly in material than the armature. Thus increasing the ampere-turns of the armature results usually in a cheapened machine, and the high densities used in gap and teeth (which demand a costly field), are due to this increase of armature strength. The question arises, then, with the densities used, what ratio of ampere-turns of the armature

to ampere-turns of the field can be allowed? The answer is obtained only from tests.

Few machines, unless compensated, show ratios exceeding unity, and general experience points to the fact that—

(1) In shunt-motors and compound dynamos above 10 K.W. with the proportions here advised, unity can always be approached if the commutator be made with a sufficient number of segments. Often it may be exceeded, 1·2 and 1·3 being obtained. But economy usually leads to a ratio of about 0·8 (compare p. 31).

(2) In shunt-dynamos, attainment of a higher ratio is complicated by the fact that heavy reaction goes with poor regulation. If this be compensated by adjusting the shunt-resistance, the same conditions hold as under (1). If not, then a ratio of 0·75 to 0·8 will give fair regulation.

This limit is largely a matter of commutation, and is consequently dependent upon the choice of the commutation constants. Evidently also it is dependent to some extent upon machine dimensions. In the smaller machines of $\frac{1}{2}$ to 4 H.P., where high saturations cannot be obtained, the ratio comes out much lower, the field ampere-turns per pole being often twice, and even more than twice, as great as the armature ampere-turns per pole.

Senstius,* who records some experiments on this subject, considers that the pole-pitch materially affects the safe value of the above ratio. He states firstly that the maximum pole-pitch should not exceed 23 inches, and that for pole-pitches varying from 15 inches to 23 inches the maximum ampere-turns per pole for satisfactory designs vary from 5000 to 6500 with average air-gap lengths, the lower figure corresponding to the smaller pole pitch, and *vice versa*. It will be seen that this may also be expressed in terms of the "specific electric loading," *i.e.* in terms of the ampere-conductors per inch of armature periphery; the above values then correspond to 750 and 600 ampere conductors per inch respectively for pole pitches varying from 13 inches to 23 inches. It is interesting to compare these with the corresponding values referred to on pp. 22 and 177.

Armature Reaction in Neutralized Machines.—The value of the armature strength where special devices are used to neutralize it is determined solely by two considerations, *viz.* temperature rise and economy. The discussion of the former will be found chiefly in Chap. VII., but it is of interest to call special attention to the limits given by Senstius † (p. 82), which would appear to allow a maximum of 800 ampere-conductors per inch for a rise of 25° C., or a maximum value of 1800 ampere-conductors per inch for a rise of 40° C.

As regards the latter, both theory and practice point to a value

* *Proc. Amer. Inst. E.E.*, vol. **24**, No. 6, p. 418. † *Loc. cit.*

of the armature strength considerably higher than that possible in non-neutralized machines. The increase of armature strength necessitates, of course, more copper for the neutralizing device, so that a limit is soon reached when further increase of armature strength becomes too costly. This limit occurs (so far as the author's experience shows) at about 1000 ampere-conductors per inch.*

Macfarlane and Burge † point out that this maximum armature strength is reached when the cost of material in the armature and neutralizing windings taken together is equal to the cost of the material in the shunt winding and of the effective iron and steel in the magnetic circuit. They thus arrive at the conclusion that the ratio $\frac{\text{flux per pole} \times \text{No. of poles}}{\text{armature ampere-wires}}$ should be about 300 for maximum economy. In the author's opinion this value is too low for large machines, where it should be about 400, while 300 for small machines will be found to give good results; but it is evident that the choice of this ratio (which undoubtedly largely influences the cost of the machine) is dependent partly on the relationship between the cost of the active copper and iron and the cost of corresponding bed-plates, end-plates, etc., which vary in every works. Thus for safe designing the value of the best ratio must be determined individually by every manufacturer, but the author believes that when once known it is one of the best guides, especially in indicating relative cost.

* Cf. Page and Hiss, *Journal I.E.E.*, vol. xxxix. p. 575.
† *Journal I.E.E.*, vol. xlii.

CHAPTER VI

TEMPERATURE-RISE—FIELD COILS

THE power-losses which attend the conversion of energy from one form to another manifest themselves generally as heat. The several parts of the machine in which these heat-losses occur have already often been referred to, and it is now necessary to show in what manner the output of a design is limited by the consequent temperature-rise. Naturally the rise of temperature of any part above that of the surrounding air will depend upon the amount of loss in that part, as also upon the amount of heat which the part can liberate or dissipate. The extreme importance of this question is now accentuated through the use of neutralizing coils (as with interpoles), which, by rendering commutation possible at almost any load, remove what has hitherto been one of the chief limiting factors in dynamo design. In consequence, where such neutralization is adopted the over-all dimensions are almost entirely dependent upon and determined by the temperature-rise of the various parts. We shall therefore go into this matter rather fully.

Causes of Heat.—The sources of power-loss occurring in direct-current machinery are as follows:—

(*a*) In the Field Magnets—

(i.) Copper loss in the windings.

(ii.) Eddy-current loss in the pole-pieces in the case of machines with toothed armatures.

(*b*) In the Armature—

(i.) Copper loss in the armature winding.

(ii.) Eddy-current loss in armature bars. This loss is usually quite small and almost impossible to estimate.

(iii.) Commutator losses, comprising copper loss due to contact resistance of brushes, brush friction loss, loss due to eddy-currents in commutator bars, and loss due to sparking (see pp. 131, 133).

(iv.) Iron-losses in the armature core, especially in the teeth of toothed armatures. These comprise hysteresis

and eddy-current energy losses, and methods of estimating them have already been given on pp. 29–30.

(v.) Windage loss of armature and friction loss of bearings (p. 27).

The above power-losses determine the quantity of heat generated in the field magnets and armature, and, with the exception of (v.), vary with the temperature.

Cooling Factors.—The causes which contribute to the cooling of the machine are (*a*) conduction, (*b*) convection, and (*c*) radiation. The dissipation of heat by conduction is effected by the iron core, the copper conductors, and the insulating material, and evidently depends on their ability to conduct heat. In the process of convection (which will also include the conduction of the air) the amount of cooling will depend upon whether the machine is at rest or rotating, open or enclosed. The most important factor in the cooling of the machine is that due to the radiating surfaces—that is, to the superficial areas of the conductors in direct contact with the air.

Laws connecting Heating and Cooling.—Though many ex-

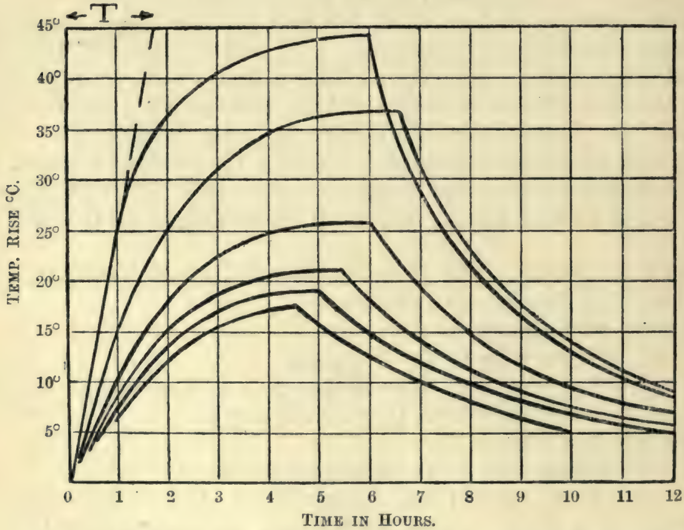


FIG. 40.—HEATING AND COOLING CURVES OF A 15 H.P. D.C. MOTOR FOR DIFFERENT LOADS.

periments have been performed to investigate the laws of radiation, yet our knowledge of the heat lost in this way is very scanty, and with regard to convection we are in almost complete ignorance; so that we have only approximate solutions to the various conditions.

Dulong and Petit's exponential * law seems to apply with better accuracy through a wider range of temperature-difference than that of Newton, and from the former it follows that the loss by radiation is proportional to—

(1) The size of the radiating surfaces of the cooling body.

(2) The excess temperature of the body above the surrounding air.

(3) The power of emission of the surface, *i.e.* the nature of the surface.

Final Temperature.—When a dynamo or motor is loaded, the generation of heat due to the power losses in any part causes the temperature of that part to increase, and as this takes place heat will be emitted in the three ways indicated. The temperature continues to rise until a maximum is reached, since the rate of emission increases with the temperature rise. The temperature becomes stationary when the rate at which heat is generated is equal to the rate of its dissipation.

Intermittent Loading.—If we had to deal with the heating of a homogeneous body which was simultaneously radiating heat uniformly over its whole surface, the time taken for a particular temperature rise would be given by the following equation :—

$$\theta = \theta_m\left(1 - \epsilon^{-\frac{t}{T}}\right)$$

where θ = temperature rise at any time t;

θ_m = maximum or stationary temperature rise;

T = time constant depending on the thermal capacity of the body.

The rise of temperature θ follows an exponential law as shown in Fig. 40. The value of the time-constant is obtained by differentiating θ with respect to t, thus—

$$\frac{d\theta}{dt} = \theta_m \cdot \frac{1}{T} \cdot \epsilon^{-\frac{t}{T}}$$

If $t = 0$, $\frac{d\theta}{dt} = \frac{\theta_m}{T}$; hence, since $\frac{d\theta}{dt}$ is the initial rate of heating, T represents the time which would be taken for the body to attain a temperature rise θ_m if no emission of heat took place.

If the supply of heat were stopped and the homogeneous body allowed to cool, the fall of temperature θ' at a time t would be given by—

$$\theta' = \theta_m \epsilon^{-\frac{t}{T}}$$

* Dulong and Petit's law states that the rate of cooling—

$$\frac{d\theta}{dt} = A(a^{\theta_t} - a^{\theta_0})$$

where θ_t = temperature of body;

θ_0 = temperature of surrounding medium, and A and a are constants.

Though the armature of a dynamo is by no means homogeneous, the actual heating and cooling curves of the direct-current motor given in Fig. 40 show that the machine still follows practically an exponential law in the matter of heating and cooling. And it is possible from tests to deduce constants for a given size of machine which, when used to modify the formulæ just derived, enable the designer to predict with considerable accuracy the temperature rise on intermittent loads—a most important matter in crane-rated motors.

Rating for Continuous Working.—It is generally found that the hottest parts of either generators or motors of modern type up to 250 kilowatts attain their maximum temperature within the limits of a six-hour run under continuous working with full load. In Fig. 40 the maximum temperature of the 15-h.p. direct-current motor with full load applied continuously is reached in about 5 hours.

This consideration has led the Engineering Standards Committee to issue the following recognized rating for continuous working:—

(A) The output of generators and motors for continuous working shall be defined as the output at which they can work continuously for *six hours* and conform to the prescribed tests,* and this output shall be defined as the Rated Load.

Rating for Intermittent Working.—In continuous working of direct-current machinery at any given load, the heating and cooling curves of the various materials employed have little influence on the final temperatures reached in any part, but they affect the time taken to reach those temperatures. And, moreover, since the different parts of a machine are not homogeneous, the heating and cooling of a particular part will be affected by the heating and cooling curves of an adjacent part of different material. In the case of motors running intermittently, such as crane- and lift-motors (which work for short periods and then stand still so that their field coils and armature cool down during the much longer periods between each run), the final rise of temperature will evidently depend on the heating and cooling curves of the materials of the various parts. Let us consider such a motor to be fully loaded for, say, t minutes and then to rest for t' minutes. During the first, the heating period, the machine will rise in temperature to a certain value along its heating curve; it then falls in the next period along the cooling curve for this temperature. If these periods be repeated successively, the temperature of the machine rises by zigzag amounts, the increments during the heating periods becoming smaller and the decrements during the cooling periods larger. Hence the machine attains a limiting temperature

* These tests are embodied in the temperature rise standards suggested by the Standards Committee on p. 17 of the Report.

when the temperature-increase during the time of load is equal to the temperature-fall during the time of standstill.

This *limiting temperature* will evidently be smaller than when the motor works continuously, and will depend on the lengths of the periods t and t' as well as the cooling properties of the materials used. The time taken to reach this temperature will also depend on these factors.

It is clearly difficult to define what shall constitute the output for intermittent working. The Standards Committee have, meanwhile,* adopted the following definition of intermittency:—

(B) The output of motors for intermittent working shall be the output at which they can work for *one hour* and conform to the prescribed tests, and this output shall be defined as the Rated Intermittent Load.

Heating of Coils.—The factors which determine the temperature rise of a coil have been shown above to depend (1) upon the power losses in the magnet coil; (2) upon the size, shape, and nature of its surface; (3) upon the ventilation of the coil, *i.e.* upon the method of support, whether closely fitting to the poles, or mounted on a metal former where there is an air-space between the core and the coil, or provided with an air space in the middle of the winding; (4) upon the load of the machine which heats the armature, and thence by conduction the magnet-core, and by convection the pole-pieces and surrounding air; (5) upon the speed of the machine, an increased circulation of air producing better ventilation.

Investigations by Neu Levine and Havill,† and recently at the National Physical Laboratory,‡ have been made to determine how the mean temperature rise depends upon some of these factors, and to find the general relationship between the maximum temperature in the interior of any coil and the mean temperature of the coil.

The mean temperature of a coil can be measured accurately by the increase of resistance of the copper composing the winding.

The rise in temperature is given by the following formula:—

$$\left.\begin{array}{r}\text{Temperature rise in}\\ \text{degrees Centigrade §}\end{array}\right\} = (238 + t)\left\{\frac{\text{resistance (hot)}}{\text{resistance (cold)}} - 1\right\}$$

* The test is under discussion with a view to revision.

† *Electrical World and Engineer*, July 13, 1901, p. 56.

‡ Report of the Engineering Standards Committee on Temperature Experiments on Field Coils of Electrical Machines, February, 1905.

§ If R_0 = the resistance of the coil at 0° C,

R_{T_1} = ,, ,, T_1° C. (the initial state of the coil, *i.e.* cold),

R_{T_2} = ,, ,, T_2° C. (the final state of the coil, *i.e* hot),

$$\text{Then } R_{T_1} = R_0(1 + \alpha T_1)$$
$$\text{and } R_{T_2} = R_0(1 + \alpha T_2)$$

where α is the temperature coefficient of copper, viz. 0·00428. It represents the

where t = temperature of the room as given by a mercury thermometer in degrees Centigrade at which the resistance (cold) is measured.

The maximum temperature in Neu Levine and Havill's experiments was obtained from the measurement of the increase of resistance of different sections of the coil, and in the National Laboratory's results by means of thermocouples inserted during the winding of the coil.

The results of the former on four different coils show that *the ratio of temperature-rise of the hottest part to the mean temperature-rise was* 1·21 *when the machine was at rest, and* 1·12 *when running at full load. The National Laboratory's results show that the difference between the temperature of the hottest part and the mean temperature varies from about* 25° *C. downwards.*

Temperatures were also taken by the National Physical Laboratory by means of a thermometer placed on the outside of the coils, and the results show that the temperature rise varies with the amount of packing and local heating so produced. On working out the National Physical Laboratory's results, the ratio of the mean temperature rise to the rise by thermometer is found to vary as follows:—

For taped coils, machine light	1·7–2·3
„ „ „ „ loaded	1·9–2·5
„ varnished coils, machine standing still . . .	1·4–1·8
„ „ „ „ loaded	1·8–2·2
„ coils with taping removed, machine light . .	1·2
„ „ „ „ „ loaded . .	1·4

The excess of the maximum over the mean temperature varied according to the shape of the coil and the temperature at which it was run. A large cooling effect on the core side was noticeable in the case of some of the coils wound on a metal former, especially where there was an air space between the core and the coil; and in the latter cases as low a temperature was obtained on the core side as on the outside, when the machine was running.

increase in resistance per ohm of the coil for one degree Centigrade rise in temperature.

$$\therefore \frac{R_{T_2}}{R_{T_1}} = \frac{R_0(1+\alpha T_2)}{R_0(1+\alpha T_1)} = \frac{1+\alpha T_2}{1+\alpha T_1}$$

$$\therefore \frac{R_{T_2}-R_{T_1}}{R_{T_1}} = \frac{1+\alpha T_2-(1+\alpha T_1)}{1+\alpha T_1} = \frac{\alpha(T_2-T_1)}{1+\alpha T_1}$$

$$\therefore \left(\frac{1+\alpha T_1}{\alpha}\right)\left(\frac{R_{T_2}}{R_{T_1}}-1\right) = T_2 - T_1$$

$$\left(\frac{1}{\alpha}+T_1\right)\left(\frac{R_{T_2}}{R_{T_1}}-1\right) = \text{rise in temperature.}$$

This is further borne out by E. Brown's * observations on a bipolar Siemens dynamo. He notes the following points:—

(1) The bobbin flanges have an influence on the cooling, and should be made of as good conductors of heat as is possible consistently with their insulating properties.

(2) The magnet-core is efficacious in promoting cooling, hence any layers between the coil and core should be good conductors of heat.

(3) The highest temperatures are in the middle of the coil, and it should be made less deep there.

In the National Laboratory's experiments, two similar coils, having the same number of watts expended in each, but wound with different gauges of d.c.c. wire, showed an appreciably lower temperature rise for the coil with the smaller amount of cotton, the coils being suspended in air.

Berrited wire produces a considerable reduction in the temperature rise, as does also little or no covering on the coil. The following temperatures were observed on two similar coils, one wound ordinarily and covered with a layer of empire cloth and a layer of varnished tape, and the other wound with berrited wire and no covering:—

	With covering.	Without covering and berrited wire.
Maximum temperature above air	80° C.	54·9° C.
Mean " "	63·6° C.	46·9° C.

Limiting Temperatures for Coverings.—Experiments seem to show that the limiting temperature for cotton-covered wire is about 125° C., at which cotton begins to darken; though up to 180° C., when it is nearly black, it is still, from the electrical point of view, a good insulator as compared with cotton at atmospheric temperature.

Temperature-rise Standards.—The Engineering Standards Committee suggest the following temperature-rises for machines in which cotton, paper and its preparations, linen, vulcanite, and similar insulating materials are used:—

Stationary coils (by resistance)	60° C. (108° F.)
Moving coils (by resistance)	60° C. (108° F.)
Moving coils (by thermometer or thermo-couple placed in contact with coil or core, whichever be the hotter)	50° C. (90° F.)

These temperature rises are based on the assumption that the air temperature is not greater than 25° C. (77° F.). If the air temperature is greater, then each of the temperature rises specified must be

* *Journal of Inst. E.E.*, xxx. 1159, 1901.

decreased by one degree for each degree difference between the room temperature and 25° C.

In all cases the temperature-rise observed during actual service of the machine must be such that the mean temperature of the machine shall not exceed **85° C.** (185° F.).

Where the insulation consists of special materials to resist high temperatures, and also where cotton, paper, linen, etc., are used as vehicles for varnishes and enamels, the permissible temperature rise depends on the properties of the insulating materials and the method of construction.

Heating.—For a given coil the temperature-rise is found to be practically proportional to the watts dissipated in it.* If we desire to employ an empirical rule connecting the coil-surface with the assigned limiting temperature rise, it will be convenient to employ a coefficient C_h, called the heating coefficient, or specific temperature increase.

Then if P_m = watts wasted in the magnet coil,
A_m = cooling surface of the coil,
T = mean temperature rise assigned,

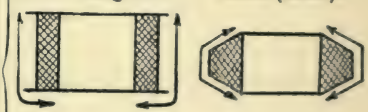
$$T = C_h \times \frac{P_m}{A_m}.$$

The fraction $\frac{P_m}{A_m}$ may be termed the specific power-loss, *i.e.* the watts dissipated per unit surface. If the specific power-loss is unity, then $C_h = T_1$; that is, the heating coefficient is the temperature increase when one square inch dissipates one watt. The coefficient C_h will depend on the factor we have already discussed when considering the radiation of the coil, and it is interesting to compare the values already given by various authors.

At the very outset one is struck with the want of decision on points which would seem to be of the greatest importance. Thus, how the temperature is to be measured, how the coil is shaped and covered, or how the cooling surface for the formula is to be estimated, are details little discussed and often left entirely to the reader's discretion. We may, however, roughly tabulate the directions and constants as follows:—

* See p. 8, N.P.L. Report, No. 19.

TABLE V.

	30° C. Thermometer.	50° C. Resistance.	Surface.
Esson * . .	0·55	—	Exposed.
Kapp . .	0·75	—	Exposed to air.
Wiener † .	—	0·68	Exposed surface and flanges if air has access to them.
Thompson ‡	—	0·67	Exposed surface, not including flanges or internal surfaces.
Oerlikon §	—	0·45	Ditto.
Goldschmidt ‖	—	0·21	Total surface, interior and exterior.
Hobart ¶ .	0·67 (average)	—	Cylindrical surface.
Neu Levine and Havill **	—	0·45	Cylindrical surface, with special allowances for coils near iron.
Arnold †† .	—	0·43 to 0·65	Long. Short (thick).  Areas chosen.

It will be seen that on the whole Arnold's are the most explicit directions, but even these leave a large margin for discretion.

Now, this margin may be much reduced by reference to recent experiments, and the author, in collaboration with Mr. J. Lustgarten, has made a careful comparison of such as he has been able to find, the most important contribution being that of the National Physical Laboratory.

* S. P. Thompson, "Dynamo-Electric Machinery," vol. 1, p. 182. 1904.
† Wiener, "Dynamo-Electric Machines," p. 369. 1902.
‡ S. P. Thompson, "Dynamo-Electric Machinery," vol. 1, p. 184. 1904.
§ *Ibid.*, p. 183.
‖ *Journal Inst. E.E.*, vol. **34**, p. 665. 1905.
¶ Hobart, "Continuous-current Dynamo Design," p. 90. 1906.
** *Loc. cit.*, p. 67.
†† "Die Gleichstrommaschine," vol. 1, p. 520.

This comparison leads to the following conclusions :—

1. *That the surface for cooling to be taken into account should include the whole surface,* i.e. *the external cylinder, the internal cylinder, and both ends.*
2. *That these surfaces, however, are not all equally valuable as radiators.*
3. *That the surfaces of the coil in direct contact with free air and those next to metal flanges in direct contact with free air are the best for dissipating heat, and are about equally valuable from this point of view.*
4. *That coil surfaces next to the iron of the pole or yoke, or next to a former fitting the pole, or so close to the yoke that no air-current is available are about half as valuable as surfaces such as those under* (3).
5. *That surfaces spaced apart from the pole or yoke, so as to allow a free air-current between them, have a value intermediate between* 3 *and* 4.
6. *That different values of C_h must be chosen according to the depth of the coil winding and the taping and insulation of the coils.*

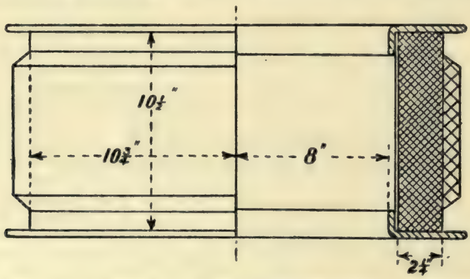


FIG. 41.—FIELD-COIL ON SHEET-METAL FORMER.

These considerations lead the author to recommend the following constants for the estimation of temperature rise of field coils.

In the formula—

$$T^\circ = C_h \times \frac{P_m}{A_m}$$

A_m is estimated for the whole coil, and is the sum of—

All surfaces included under (3);
Half the surfaces included under (4);
From 0·7 to 0·8 of the surfaces under (5).

The values of C_h then are—

A. (Figs. 41 and 42) *For coils on metal formers with no external taping, and a depth not exceeding* 3″—

$$C_h = 130 \text{ to } 150$$

$C_h = 130$ is to be used only where the coil is well below 3" in depth; 150 is a good safe value.

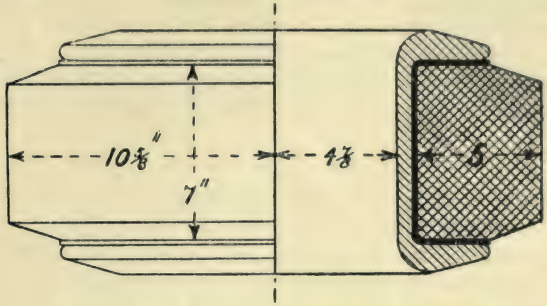


FIG. 42.—FIELD-COIL ON CAST-IRON FORMER.

B. (Fig. 43) *For coils with two or three layers of tape around them*—

$$C_h = 200$$

If impregnated and taped, a somewhat lower value may be taken, as—

$$C_h = 180$$

C. *For coils wrapped in canvas, fullerboard, etc., up to a thickness of ⅛" all over*—

$$C_h = 220$$

Where an exceptional depth of coil is used, as 5", C_h must be increased slightly, usually 10 per cent. to 15 per cent. for each inch of depth above 3".

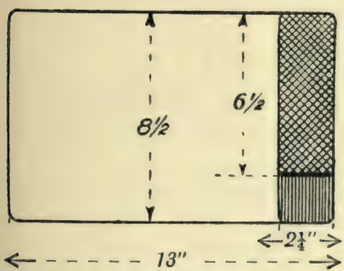


FIG. 43.—TAPED-COMPOUND COIL.

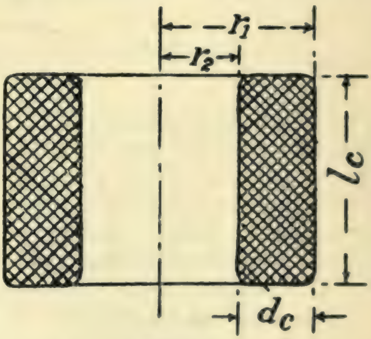


FIG. 44.

T is from 15° to 25° C. higher than the temperature measured by thermometer.

Deductions from the Results.—The results just summarized

are remarkable in many ways, but especially in pointing the advantage to be gained by proper ventilation of the field coils. It is very clear that leaving room for air circulation about a coil will increase by no inconsiderable amount the watts which that coil can dissipate (for a given temperature-rise). This advantage will depend upon many factors, but chiefly upon the relationship between the depth of the winding (d_c in Fig. 44) and the internal diameter of the coil ($2r_2$ in Fig. 44).

Now—

$$2r_2 = \text{diameter pole} + \text{clearance for air circulation}$$
$$= \text{diameter of pole} + 2(\tfrac{3}{4}''), \text{ usually,}$$

so that the advantage to be gained will depend very largely on the relationship between the diameter of pole and depth of coil.

Depth of Coil.—The depth of coil which is found to be of use in practice is curiously constant. In modern machines, it only varies

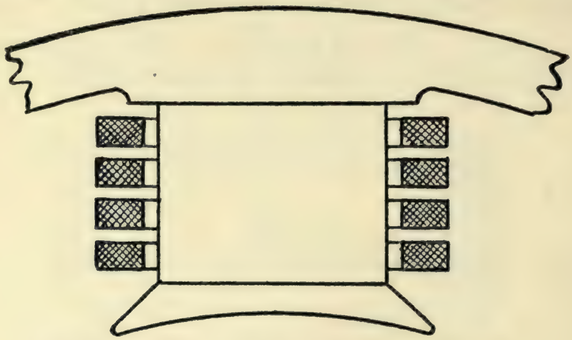


FIG. 45.—FIELD-COIL DIVIDED FOR VENTILATION.

from 2″ in machines of 4 H.P. up to 4″ (or very rarely 5″) in machines of the largest size; and since the diameter of the pole is by no means so constant, there will be a size of machine below which it does not pay to insert these ventilating spaces. Naturally this limit depends largely upon many variables, not the least of which is the price of copper, so that it is difficult to fix even approximately; but the author would place it at about 30 kilowatts.

In machines of 80 kilowatts or more, it becomes economical to break up the field coil into a number of sections as shown in Fig. 45.

These may be from 1″ to 2″ in length, with air spaces of $\frac{3}{4}''$ to 1″ all round. Fig. 93 shows a method devised by the author for fixing insulating and ventilating compound-wound field coils.

General Formulæ.—From the summary on pp. 72, 73, for any given shape of coil general and particular formulæ may be easily derived.

CASE I. The upper annular and inner cylindrical surfaces of the coil in Fig. 44 are supposed spaced apart from the yoke and pole respectively, and fall under paragraph 5 on p. 72. Then if the pole be circular in section, and k is the fraction of the surfaces not fully exposed to air, considered as effective under these paragraphs—

$$A_m = \pi d_c^2(1 + k) + 2\pi r_2(l_c + d_c)(1 + k) + 2\pi d_c l_c$$

with $k = 0{\cdot}75$ this becomes—

$$A_m = 5{\cdot}5 d_c^2 + 11 r_2(l_c + d_c) + 6{\cdot}28 d_c l_c$$

If the pole be square in section, then in Fig. 44 $r_2 = (\frac{1}{2}$ width of one side of pole + clearance), and

$$A_m = 8l_c\{d_c + r_2(1 + k)\} + 4d_c(d_c + 2r_2)(1 + k)$$

Or if $k = 0{\cdot}75$

$$A_m = 7d_c^2 + 8l_c d_c + 14r_2(l_c + d_c)$$

CASE II. The field coil is supposed to fit pole and yoke closely. Here $k = 0{\cdot}5$, so that for circular poles the equation is—

$$A_m = 4{\cdot}7d_c^2 + 9{\cdot}4r_2(l_c + d_c) + 6{\cdot}28 l_c d_c$$

For square poles it is—

$$A_m = 6d_c^2 + 8l_c d^c + 12r_2(l_c + d_c)$$

CASE III. *Coil divided into sections* * (Fig. 43). The lowest section may be taken under Case I.

For each of the other sections with circular pole—

$$A_m = 2\pi(r_2 + d_c)l_c + k\{2\pi r_2 l_c + 2\pi(2r_2 + d_c)d_c\}$$

If $k = 0{\cdot}75$—

$$A_m = 4{\cdot}7d_c^2 + 9{\cdot}4r_2 d_c + 11r_2 l_c + 6{\cdot}28 d_c l_c$$

With square pole—

$$A_m = 8l_c(r_2 + d_c) + k\{8r_2 l_c + 8(2r_2 + d_c)d_c\}$$

If $k = 0{\cdot}75$—

$$A_m = 6d_c^2 + 12r_2 d_c + 8l_c d_c + 14l_c r_2$$

Often the value of d_c can be approximately fixed, and when this is so the foregoing equations give the values for l_c, because C_h, T, and P_m are all known from the circumstances of the case.

Example.—We are now in a position to illustrate these calculations by completing the field-coil which was calculated on p. 59. We shall start this time from the supposition (generally true for new machines) that the watts to be dissipated are known roughly from the desired efficiency, and that the temperature rise is fixed.

We shall suppose that the ampere-turns to be provided per pole are those previously calculated, viz. about 7000 for the shunt, and

* In which case l_c refers of course to the length of *one section*.

3800 for compounding. Further, that the watts allowable for each shunt coil are about 275, which with a voltage of 190 across the 4 coils corresponds to $C_f = 5{\cdot}7$ amperes.

The watts to be expended in each series coil are 150; and the series current is 480 amperes, corresponding to 8 turns per pole. The series coil is to be placed nearest the pole-tip, and the shunt coil is to be divided (if necessary) into two or more sections to secure the requisite cooling surface. The coils are all to be taped and impregnated; $T = 60°$, $C_h = 180$. It will be of interest first to examine such an arrangement as that already calculated on p. 59, to see if it can fulfil the stipulated conditions.

Considering first the shunt coil alone, we have—

$$A_m = C_h \frac{P_m}{T} = 180 \cdot \frac{275}{60} = 825$$

Applying Case II., taking $r_2 = 6''$

$$A_m = 4{\cdot}7d_c^2 + 6{\cdot}28l_cd_c + 56{\cdot}4(l_c + d_c)$$

So that—

$$l_cd_c = \frac{825 - 4{\cdot}7d_c^2 - 56{\cdot}4(l_c + d_c)}{6{\cdot}3} \quad . \quad . \quad . \quad (1)$$

Now, this equation connects l_c and d_c; and a second equation connecting the same two quantities may be obtained, as is shown in Appendix IV., p. 230.

If we remember that the specific resistance must be that of the material when hot, say $\frac{82}{10^8}$ ohms, and that the length of mean turn will be $\pi(2r_2 + d_c)$, we obtain from the second equation—

$$l_cd_c = \frac{4}{\pi} \cdot \frac{82 \times (\text{ampere-turns per coil})^2 \times \pi(2r_2 + d_c) \times m^2}{10^8 \times \text{watts per coil}}$$

where m is the ratio $\dfrac{\text{diameter of wire covered}}{\text{diameter of wire bare}}$.

The diameter of the wire is known to be in the neighbourhood of No. 14 d.c.c. (cf. p. 59), for which from any wire tables m^2 is found to be about 1·4.

Thus—

$$l_cd_c = \frac{82 \times 7000 \times 7000 \times \pi(12 + d_c) \times 1{\cdot}4}{10^8 \times 285} \quad . \quad . \quad (2)$$

If we take the value of d_c for which the coil was originally arranged, viz. $2\frac{1}{4}''$, we find from these equations $l_c = 10''$, which with a pole of $8''$ is manifestly impossible.

If, on the other hand, we take the maximum permissible depth consistent with reasonable temperature in the coil centre, viz. $4''$, we find, when the series winding is considered, l_c = more than $9''$, which is still impossible.

Thus an unventilated winding such as that proposed in Chapter V. is impossible from the point of view of temperature rise.

Calculation of Ventilated Coil. Shunt Winding.—Suppose now that we space the shunt-coil 1″ from the pole all round, and divide it into two sections. Then each section may be taken under Case III., and below the two shunt-coils, if there is room, we can put the series turns.

Evidently each section of the shunt-coil must dissipate $\frac{275}{2} = 142\frac{1}{2}$ watts; and the length of mean turn will be $\pi(14 + d_c)$.

Also $$A_m = 180 \times \frac{275}{60 \times 2} = 412 \text{ sq. ins.}$$

* Equation (1) is, from Case III.—

$$412 = 4{\cdot}7d_c^2 + 66d_c + 77l_c + 6{\cdot}28d_cl_c \quad . \quad . \quad . \quad (1)$$

* Equation (2) is—

$$l_cd_c = \frac{82}{10^8} \times \frac{(7000)^2}{4} \times \frac{\pi(14 + d_c) \times 1{\cdot}4}{142{\cdot}5}$$
$$= 4{\cdot}4 + 0{\cdot}31d_c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

whence we obtain:—

$$d_c = 3'', \ l_c = 1{\cdot}8''$$

If we allow a space of $\frac{3}{4}''$ between yoke and coil for air, and $\frac{3}{4}''$ between one section of the coil and the next, we shall have—

$$\text{Length of shunt-coil} = 0{\cdot}75 + 1{\cdot}8 + 0{\cdot}75'' + 1{\cdot}8''$$
$$= 5{\cdot}1''$$

Series-winding.—Taking now the series-coil under Case I., and of the same depth as the shunt, we have—

$$450 = 5{\cdot}5 \times (3)^2 + 11 \times 7(l_c + 3) + 6{\cdot}28 \times 3l_c$$
$$A_m = 180 \,.\, \tfrac{150}{60} = 450$$

whence $l_c = 2''$ practically; and this allows of ventilating spaces between shunt- and series-coils.

Thus with an unventilated coil the shunt-winding *alone* will hardly go in, whilst with ventilation there is room for both shunt- and series-coils.

Size of Wire.—The turns per section of the shunt-coil will be—

$$\frac{7000}{5{\cdot}7 \times 2} = 623 \text{ about}$$

and since $m^2 \times (\text{diameter of wire})^2 = \dfrac{l_c \,.\, d_c}{623}$

$$\text{diameter of wire} = \sqrt{\frac{1{\cdot}8 \times 3}{623 \times 1{\cdot}4}}$$
$$= 0{\cdot}0787$$

which is practically 14 S.W.G.

* These pairs of equations lead to one of cubic form, difficult to solve. Usually the best way of dealing with them is by means of squared paper.

The current-density is about 900 amperes per square inch, and the size of the series-winding can be calculated as shown on p. 60.

Notes on the above Calculations.—The method of field-coil calculation just outlined is, so far as the author is aware, the first to attempt any rigid connection between the four quantities, ampere-turns, watts lost, coil-dimensions, and temperature-rise. If used with discretion it will be found to be reliable, but it is naturally open to various errors, and the reader should be alive to these. In the first place, the equation from which l_c is derived is of the form—

$$l_c = \frac{A - B}{C}$$

which is always unsatisfactory, since when A and B are large and comparable, a small percentage change in either will alter the value of l_c so much.

Next the value chosen for C_h naturally affects the results considerably, and some experience is necessary to judge the conditions likely to affect the choice. These include effect of armature-windage, shape of field, amount of protection, and clearances allowed in the case of ventilated coils.

Cost of Ventilated Coils.—It is, of course, obvious that the amount of copper involved in the ventilated coils with an 8-inch pole is greater than that required for a close-fitting coil with a longer pole. The latter, however, involves greater weight of material in poles and yoke, so that only by a careful comparison of the cost of material in the two cases can the most economical design be arrived at.

CHAPTER VII

TEMPERATURE-RISE—ARMATURES AND COMMUTATORS

Heating of Armatures.—The losses which occur in the armature causing and resulting in "temperature-rise" have been enumerated on p. 63. It may here be pointed out that great differences are often found between the calculated and the actual values of the iron losses. This lack of agreement depends to a great extent on the amount of burring over of the discs, and will be more marked at the higher inductions; by careful milling or filing it can be considerably reduced. (See footnote, p. 204.)

The amount of heat liberated from the armature will depend chiefly on the heat-radiating surface. In estimating this surface we shall have to consider the influence of the core and the spider. The core, though much of it is covered with insulation, is a good conductor of heat; and the inner surfaces, including the spider, offer great facilities for dissipation of heat where a good circulation of air is obtained. This, for example, is the case where the armature is provided with three or four ventilating ducts, as in Figs. 3 and 110. The end-connections in former-wound armatures are arranged to allow of a good circulation of air to the spider, and also have an excellent fanning action.

A. H. and C. E. Zimmerman * found that as the peripheral velocity of the armature is increased, the amount of heat liberated per degree rise in temperature is also increased; but that the rate of increase becomes less with the higher speeds. The influence of the peripheral velocity on the cooling will depend chiefly on the construction of the armature-core, the arrangement of the winding, and the disposition of the field-poles. The latter tend to prevent the radiation of heat; and as the percentage polar embrace increases, the amount of heat radiated per degree rise in temperature becomes less.

The rise in temperature of the armature will also be influenced by the ventilation of the room in which the machine is placed, so

* A. H. and C. E. Zimmerman, *Trans. Am. Inst. Elec. Engs.*, vol. x. p. 336 (1893).

that a machine standing in a draughty place usually has a low temperature, since cooling by convection is more effective than by radiation. It is for this reason that fans have recently been introduced (Fig. 124) in motors to cause a definite current of air through the armature.

Measurement of Armature Temperature.—The end-connections (both front and back) cool usually better than the part of the winding embedded in the slots,* and since the length occupied by these connections is sometimes as much as three times that of the armature core, it would be scarcely fair to estimate the temperature of the latter by the "increase of resistance" method. As the induction is greatest in the teeth of the armature, the iron losses also will be greater there, and the maximum temperature will be registered better by a thermometer. In testing armatures for temperature-rise after the run, thermometers are required for the winding, for the armature iron, and for the commutator. They are held in place by means of small pieces of cotton-waste wedging their backs (which are sometimes covered with tin-foil) against the surface. These pieces of waste, by reason of their low conductivity, prevent the radiation of heat from the thermometer when its temperature is above that of the surrounding air. The thermometers are left in place until their readings begin to decrease, usually from 10 to 15 minutes. The highest temperature is generally found between two ventilation ducts in the middle of the armature. The temperature thus actually measured is higher than that which ever occurs when the armature is running, because directly rotation ceases the dissipation of heat is very much reduced.

Methods of estimating Temperature Rise.—As in the case of field-coils, so again here authors are often quite indefinite as to the surface to be used for estimation. It is, however, universally admitted that an expression of the form—

$$T = \frac{a}{1 + bv} \times \frac{P}{A}$$

gives the best approximations. In this formula a and b are constants, v is the linear circumferential velocity of the armature in feet per minute, P is the number of watts to be dissipated, and A the area of the surface of the armature in square inches. T is in degrees Centigrade.

There is some difficulty in properly estimating the quantity P, but the real trouble is in deciding upon what basis A shall be computed.

The Value of P.—This is usually obtained by adding together the watts lost in the armature copper and iron. The value of the iron-loss watts has been dealt with on pp. 29, 30, and the total copper-losses are easily obtained from the value of the armature resistance.

* Cf. Method 5, p. 82.

Professor Arnold * has, however, pointed out that the copper-losses taking place in the end-connections cannot properly be considered as heating the armature-core, and there is some justification for this view, as results testify. Thus, in computing the armature-heating by his method the whole of the iron-losses are added to those copper-losses which take place in the copper actually lying in or on the core.

In criticism of this system, it may be remarked that much depends upon the ratio of iron-losses to copper-losses, and upon the ratio of copper-losses due to the copper in the slots to that occurring in the connections. These factors depend again upon various conditions, notably the number of poles; and in any case the method hardly lends itself to preliminary calculations.

The Value of A.—The number of ways of calculating A is very large, each designer having his own system. We may distinguish two.

(1) That most commonly adopted, in which

$$A = \pi(\text{diameter of armature} \times \text{length over end-connections}).$$

(2) That which takes into consideration the ventilating spaces, etc., existing through the core. This value of A is best seen by

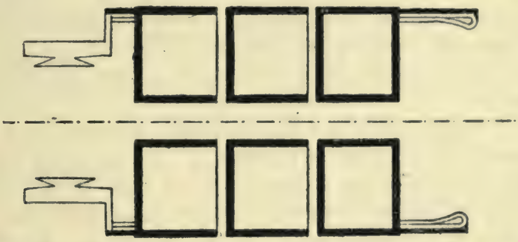


FIG. 46.—COOLING SURFACE OF ARMATURE. METHOD 3.

reference to Fig. 46, in which those surfaces marked with thick black lines are considered as effective and are added together to give A. †

Values of constants a and b.

For method (1), the value of b may be taken at 0·0005. For well-ventilated machines a then varies between 45 and 20. The former value is to be taken for the smaller machines, as, for instance, those with armatures about 12 to 18 inches diameter, where the hole through the core is largely blocked by shaft and spider. It should be added that the figures given by Hobart, ‡ calculated on this basis, would make the value of a higher; but the author finds the above reliable.

For method (2), $b = 0·0005$ and a varies between 45 and 90 for

* "Die Gleschstrommachine," vol. i. p. 527.

† Cf. Page and Hiss, *Journal I.E.E.*, vol. xxxix. p. 576; also Arnold, *loc. cit.*

‡ "Electric Motors," Whittaker & Co., 1st Edition.

well-ventilated armatures, the higher figure being for very small machines. In criticism of this system it should be said that much depends upon the width of the ducts through the stampings, which vary considerably with different makes, and in some cases are so narrow as to be practically useless.

The methods just given are a fair summary of those commonly in use, and will be found in most cases to compare well with one another. There is, however, no alternative but for the designer to determine for himself which method is most reliable for the particular type of machine he is at work upon, and by careful tabulation to build up for himself a series of consistent approximate data. Especially is this the case when a fan is added inside the motor (as Fig. 124); though roughly it is found that such a fan decreases a by 5% to 10%.

There are other methods not entirely dependent upon the formula, two of which deserve attention.

Method 3 consists in expressing the number of watts that an armature will dissipate as a function of the product (core-diameter^2 $\times$ core-length) for each peripheral speed. It is evident, since the cooling surface is not directly proportional to the quantity (diameter^2 $\times$ length), that such a relationship must be determined for each particular type of armature. Indeed, if the relationship between core-diameter and length be assumed or known, the desired curves may be deduced from methods (1) or (2). The necessity for knowing the length renders the system unsatisfactory, but it is extremely handy in another way; for if the number of watts to be dissipated by the armature is known (and this is usually obtainable from the output and efficiency), the product (diameter^2 $\times$ length of the core) is also known for each peripheral speed. Further, since the relationship between diameter and length has been shown to depend on the number of poles (p. 20), the dimensions of the armature-core as limited by temperature are known for a given output and speed and number of poles. Figs. 42 and 48 show values for the above relationship as given by Macfarlane and Burge.*

Method 4.—This method, due to Senstius,† is here given more because of its simplicity and originality than because of its general use, though it certainly leads quickly to useful limits. The system is founded on an assumption akin to that made by Arnold, viz. that the heating of the end-connections is practically independent of that of the core. Arnold, in general, deals with the core; while Senstius considers that by proper ventilation, if the frequency do not exceed 30, the core can always be kept cool; and that the limit is imposed by the end-connections, for which he gives the following rule:—

* *Journal I.E.E.*, vol. xlii. pp. 239–240.
† *Proc. Amer. I.E.E.*, vol. xxiv. p. 422.

"For peripheral speeds ranging from 1500 feet per minute to

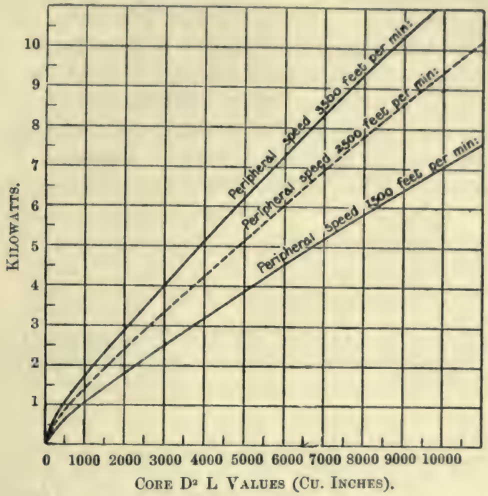


FIG. 47.—PERMISSIBLE KILOWATTS LOSS IN ARMATURE—AS A FUNCTION OF THE CORE SIZE FOR SMALL MACHINES.

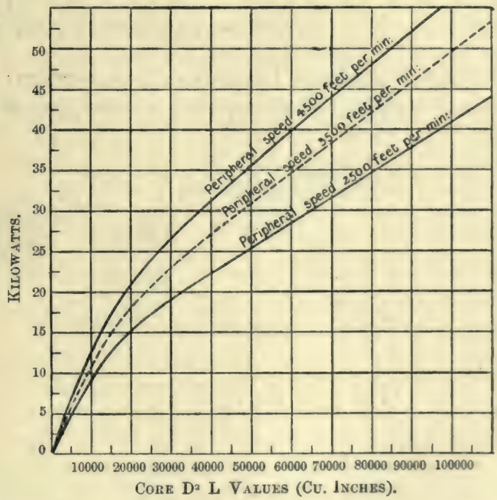


FIG. 48. PERMISSIBLE KILOWATTS LOSS IN ARMATURE—AS A FUNCTION OF THE CORE SIZE FOR LARGE MACHINES.

7000 feet per minute, for a coil-depth (not slot-depth) of 1·25 inch,

and a temperature-rise of the end-connections not exceeding 25° C., the number of ampere-turns per inch of armature-circumference varies as a straight line from 260 ampere-turns per inch to 400 ampere-turns per inch." Presumably, Senstius means in the above "coil-depth" *not exceeding* 1·25 inch, as he uses it himself in that way.

Comparison of Methods.—The results shown in Figs. 47 and 48 may be deduced direct from the formulæ of methods either (1) or (2), provided that a relationship between diameter and length of armature is assumed. Thus, suppose from p. 20 we take the ratio for poles, viz.—

$$\frac{\text{diameter of armature}}{\text{number of poles}} = \frac{\text{length of core}}{1{\cdot}5\lambda}$$

Then in applying method (1) we need but an approximate relationship between length of core and gross armature length; which may be written—

$$\left.\begin{array}{r}\text{armature-length includ-}\\ \text{ing end-connections}\end{array}\right\} \begin{aligned} &= \text{core-length} + \tfrac{3}{4}\text{ pole pitch} \\ &= \text{core-length} + \tfrac{3}{4}\pi \, . \frac{D}{p} \\ &= \text{core-length} + 2{\cdot}35 \, . \frac{\text{core-length}}{1{\cdot}5\lambda} \\ &= 2{\cdot}36 \times \text{core-length nearly.} \end{aligned}$$

For square poles, in which the figure 1·5 becomes 1·2 (p. 20), we have, armature-length including end-connections = 2·8 × core-length.

$$\text{Then in the formula } T = \frac{P}{A} \cdot \frac{a}{1 + bv}$$
$$\text{if } T = 40^\circ \text{ C.}$$

For round poles—

$$D \times L = \frac{P}{300} \cdot \frac{a}{1 + bv} \text{ roughly}$$

for square poles—

$$D \times L = \frac{P}{350} \cdot \frac{a}{1 + bv}$$

Multiplying both sides of the above equations by D gives an immediate comparison with method 3.

Example.—As a concrete case for comparative purposes, we may check the machine shown in Fig. 24 by, say, methods 1, 2, and 3. We will assume a peripheral speed of 2500 feet per minute, corresponding to about 400 R.P.M.

As the machine has 4 poles

$$\text{Length over end-connections} = 14'' + \tfrac{3}{4} \cdot \frac{\pi \cdot 24}{4}'' = 28\tfrac{1}{4} \text{ inches}$$

Cooling surface (method 1) = 2130 square inches

$a = 27$ say

so that P = 7150

Cooling surface (method 2) = 3700 square inches

$a = 54$

so that P = 6200

Again, from Fig. 47, method 3—

Diameter^2 × length = 8060

P = 7800

The third method thus gives a result approximating to method (1); while method (2) gives a much lower value. This is because the number of ventilating spaces for this length of armature is small.

With three ventilating spaces, which would be more usual, method (2) gives—

P = 7500 watts

So that this is a good check on the others, and probably the most satisfactory form for final calculations.

General Dimensions derived from Heating Formulæ.—From the relationships detailed above, it is evident that the necessary dimensions of the armature for a given output can with some accuracy be predicted; the more so if the peripheral speed be approximately determined. So obvious is this that emphasis is hardly necessary. However, for the sake of clearness, the example which has already been developed on p. 23 may be taken. It was there shown that by the formulæ which ordinarily limit the machine dimensions, a 200 K.W. 400 r.p.m. generator would require an armature with a D^2L value = 10,000. If we assume again an efficiency of 93 per cent. at $\frac{3}{4}$ full load, the variable losses will be about 5·25 K.W. at that load, or 9·3 K.W. at full load. A preliminary estimate of the iron-losses, such as that on p. 30, shows that these will amount to about 2000 watts. From the variable losses must be subtracted the loss in compounding coils, say $\frac{3}{4}$ per cent. or 600 watts; and the commutator C^2R loss. The latter has not yet been considered, but we may estimate it at 1 K.W. The balance then, viz. $(9{\cdot}3 + 2) - (1 + 0{\cdot}6) = 9{\cdot}7$ K.W., has to be radiated by the armature. From method (3), Fig. 48, assuming a peripheral speed of about 3000 feet per minute, we shall require an armature with a D^2L value of about 9000.

$$\text{From method (1) } T = \frac{a}{1 + bv} \cdot \frac{P}{A}$$

$$\text{or } A = \frac{aP}{T(1 + bv)}$$

Now, from p. 84, $A = \pi DL \times 2{\cdot}36$

Inserting this value, and also $a = 27$, $P = 9700$, $T = 40°$, $b = 0{\cdot}0005$, $v = 3000$, we obtain $DL = 300$.

Whence for a 4-pole machine in which the poles are circular, and

$$D = \frac{pL}{1{\cdot}7}(\lambda = 1{\cdot}13)$$

we get $D^2 = 710$, and $D = 27''$ nearly

Also, since $L = 11{\cdot}5$, $D^2L = 8100$

which compares fairly well with method 3.

As a further test, try the method of Senstius.

For 25° C., at 3000 feet per minute, the ampere-turns per inch will be, according to the rule given—

$$260 + \frac{3000 - 1500}{7000 - 1500} \cdot \frac{400 - 260}{1} = 298$$

For 40° C., then, they will be—

$$\sqrt{\tfrac{40}{25}} \times 298 = 377 \text{ ampere-turns per inch}$$

or ampere-conductors per inch = 754

Inserting this value in the equation on p. 22, viz.

$$D^2L = \frac{60{\cdot}8 \times \text{pole-pitch} \times 10^7}{\text{density at face} \times 754 \times \text{pole-arc}} \times \frac{\text{watts}}{\text{R.P.M.}}$$

$$= \frac{60{\cdot}8 \times 500 \times 10^7}{55{,}000 \times 754 \times 0{\cdot}7} = 10{,}500$$

which is somewhat higher than the result given by methods 1 or 2, probably because the "coil-depth" will not approach the limit of Senstius, viz. $1\frac{1}{4}$ inch. Yet allowing for this, and considering how different the methods are, the agreement must be considered very good, as providing a starting-point for a design limited only by heating.

Commutators.

Losses causing Heat.—These may be divided into (*a*) electrical, (*b*) mechanical.

The former comprise the losses due to resistance of the bars, and of the contact between the bars and the brushes. There is also a small eddy-current loss in the commutator section itself due to the changes which take place in the current it is called upon to carry. Of these losses, that due to the resistance of the brush contact is by far the largest, and the methods of calculating it, together with some account of the factors upon which it depends, are given under the heading of "Dimensions of Commutators" on p. 133.

The mechanical loss is that due to the friction of the brushes upon the commutator. This is dealt with on p. 133.

The construction of the commutator should always be such as to

provide as much cooling surface as possible. Examples of modern designs are given in Figs. 3 and 112, from which the methods of providing ventilation will be apparent.

For any construction, however, the heating of the commutator is usually estimated from the external cylindrical cooling surface only. With good ventilation for 40° C. rise, the watts which this surface will dissipate per square inch = 3 to 5; with poorer ventilation, 2 to 3 watts per square inch.

On the average then

$$\frac{\text{commutator resistance- and friction-losses}}{4} = \left\{\begin{array}{l}\text{area of commutator ex-}\\ \text{ternal cylindrical sur-}\\ \text{face in square inches}\\ \text{as limited by tempe-}\\ \text{rature rise}\end{array}\right.$$

The length of the external cylindrical surface is measured from the outside of the commutator-segment to the riser, and excludes the length occupied by the risers themselves.

Naturally, the value of the constant to be adopted in the above formula is dependent upon the peripheral speed of the commutator-face. This, in ordinary machine design, rarely exceeds 3600 feet per minute, as chattering of the brushes is liable to occur at higher speeds.

We have then a rough limit for the size of the commutator-face, and when the diameter is not limited by peripheral speed it is usually determined by the number of segments, each of which with its appropriate insulation is rarely less than $\frac{3}{16}$ inch thick at the commutator-face. The application of the above figures is so patent as to need in this place no concrete example.

Heating of Totally Enclosed Motors.—The temperature-rise of totally enclosed motors is best estimated from the total radiating surface which the complete machine offers. The method of calculating this surface varies with different designers, but in general it is taken to be the total external surface of the motor. On account of the fact that the bearings sometimes heat rather more than any other part, some designers reckon the exposed radiating surface of the bearings twice over. Within ordinary ranges of motor designs, as from 5 to 50 H.P., a temperature-rise of 50° C. will be reached when from 0·4 to 0·6 watt is dissipated for each square inch of external surface. An average figure would be 0·5. Artificial increase of the external surface by means of ribs or similar projections will modify the above figure; and it may be said in general that it is best to collect, from experimental data on the particular type of machine in hand, a series of results which can be put into curve form for future use. This is especially necessary in cases where machines are to be short-rated. It may be added that fans in T.E. motors are useless unless ribs be provided inside the case as well as outside.

Such a series is shown for a particular style in Fig. 49, and the values are fairly typical. On the curves the figures $\frac{1}{2}$, $\frac{3}{4}$, etc., refer to the hours at which the machines are rated for a rise of 50° C., and the dotted lines AB and CD represent two sizes of machine. Thus the point A shows that the machine represented by the dotted line AB will stand a load of 72 watts per revolution per minute for a rise of 50° C. in three hours, and that the watts per square inch of radiating surface are then 0·75. Having a series of limiting values

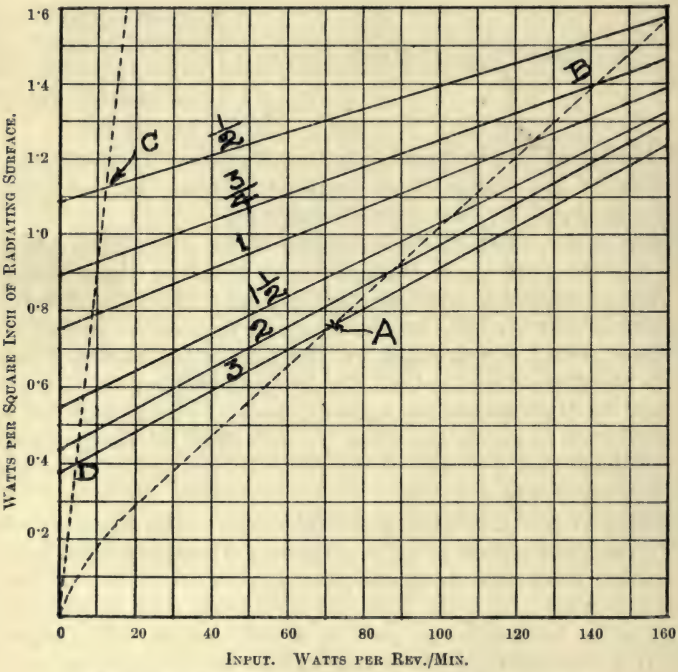


Fig. 49.—Heating of totally enclosed Motors.

like these, it is easily possible to interpolate for any new intermediate size. Thus, suppose a machine be required capable of giving 9000 watts at 300 revolutions per minute; *i.e.* 30 watts per revolution per minute. From the curve, it is evident that for a three-hour rating each square inch of the carcase will dissipate 0·53 watt. If the efficiency of the machine be estimated at 80 per cent., the input will be 11,250 watts, and the losses 2250 watts; so that the external radiating surface required is $\frac{2250}{0{\cdot}53} = 4250$ square inches.

CHAPTER VIII

ARMATURE WINDINGS

It would not be possible, nor is it desirable, in a work of this scope to describe in detail either all the elementary principles of armature winding on the one hand, or all the various forms into which those windings have ultimately developed on the other. It should be sufficient to indicate in a general way the principles underlying all the *usual* forms of armature winding, and to set down in some detail the rules governing the application and form of those windings which are now most commonly met with. It is supposed that the reader is familiar with the rudiments of armature winding, and the outline now to be given must be filled in from his previous knowledge or subsequent experience.

Types of Armature Winding.—There are two general types of armature winding—the *open-coil and the closed-coil.* In the first type current is taken from each coil only at the time when it is developing its maximum E.M.F., all the other coils being at that moment cut out entirely. The open-coil armature is only used for series arc lighting, where a constant current with a high potential difference is required; the advantage of the system lies in having no difference of potential between adjacent commutator-bars connected to coils not simultaneously in circuit. This type of armature is not much used, having been replaced almost entirely by the second type.

Closed Coil Armatures.—The closed-coil armature, apart from the commutator, forms an endless winding, so that when we imagine the commutator and core dissolved away without otherwise disturbing the winding, on pulling the latter it would be found to consist of one or more endless bands. Such a winding is said to be "re-entrant," because it closes upon itself; * one winding returning on itself is said to be *singly re-entrant,* whereas a winding formed of two or more independent endless bands, each closing upon itself, is said to be *multiply re-entrant.* Thus in a "doubly re-entrant"

* The notation here followed is that of Parshall & Hobart; see also Cramp's "Armature Windings of the Closed-circuit Type," Biggs & Sons.

armature winding we have two separate windings each closing upon itself. These short definitions are here inserted on account of the confusion that has arisen in the use of these terms.

A closed-coil armature can have no less than two paths in parallel through it. Each of these will have the same number of turns, and the current entering one brush will divide equally through the two halves to unite again at the other brush. There may be more than two parallel paths for a "singly re-entrant" winding, as we shall see later; and, of course, there will be more than two parallel paths for a "multiply re-entrant" winding.

Closed-coil armature windings can be divided into two classes, ring winding and drum winding.

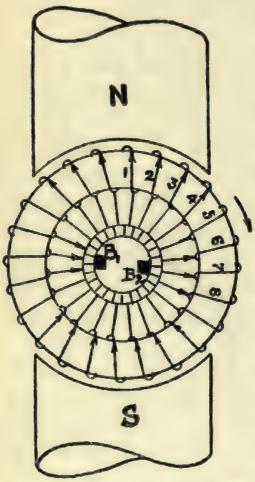


FIG. 50.—BI-POLAR RING ARMATURE.

Elementary Bi-polar Ring.—In order to make subsequent explanations more clear, we must here refer to this now practically obsolete type. The simple Gramme ring armature consists of a series of spirals of wire wound about an annular core in regular order (Fig. 50), the last turn of the last spiral being connected to the beginning of the first. At points equally spaced around the winding connections are made to commutator segments as shown in Fig. 50. In actual practice the end of each coil would be brought down to the commutator segment, and there connected to the beginning of the next coil, thus avoiding the large number of soldered T joints. With two brushes at opposite ends of a neutral diameter B_1, B_2, the winding is divided into parallel halves. If a = cross-section of the wire in square inches, and l' = total length of wire in inches, then the resistance of each half of the winding is $\frac{\rho \cdot \frac{l'}{2}}{a}$, where ρ = the specific resistance of copper at some definite temperature. Hence the armature resistance R_a is—

$$R_a = \tfrac{1}{2} \cdot \rho \cdot \frac{\frac{l'}{2}}{a} = \frac{\rho l'}{4a}$$

Turns per Segment.—Instead of one turn per commutator segment, as shown in Fig. 50, there may be as many turns between one segment and the next as is consistent with sparkless

commutation * (see Fig. 51); the author has had instances of as many as 80 turns in each armature coil, *i.e.* between one commutator segment and the next.

Multipolar Simple Ring-Winding.—The ring-armature in Fig. 50 can be employed for a machine with four poles (Fig. 51); and, in general, if the armature has a sufficiently large number of spirals, the same winding can be used with any number of poles.

The arrows in Fig. 51 show the relative directions of the various induced E.M.F.'s. In order that currents may naturally flow under the influence of these E.M.F.'s, it is obviously necessary that collection should take place at those places where each series of induced E.M.F.'s meets its neighbouring series. These places are often called "neutral points" (cf. p. 52), because no E.M.F. is generated in coils which lie there, since such coils are under the influence of no field.

The number of neutral points with four poles is four, and for electrical balance we require four brushes, alternate brushes being of the same polarity and connected to one another. The number of circuits in the armature is the same as the number of poles. The addition of another pair of poles, if that were possible, would necessitate another pair of brushes, and would add thereby another pair of armature sections, and so on. These sections or circuits are put in parallel at the brushes, and as each section has the same current-carrying capacity, the current that can be taken from the machine is proportional to the number of poles. Such windings in which the minimum number of circuits possible is equal to the number of poles are designated *multiple-circuit windings*.

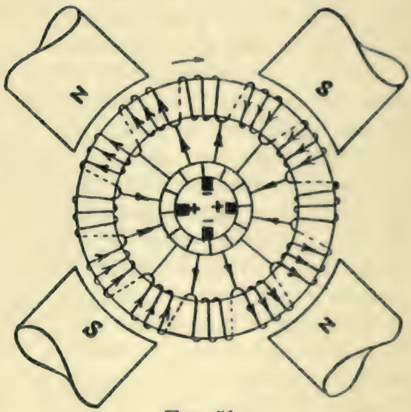


FIG. 51.

Pitch.—The *distance* from spiral to spiral in Figs. 50 or 51, *i.e.* $\frac{1}{S}$ of the total armature circumference, where S is the number of spirals or of coils, as the case may be, is called the *distance-pitch* of the spirals or coils. If, as in Fig. 50, the number of coils is 24, the *distance-pitch of the coils* is $\frac{1}{24}$. If the spirals be numbered consecutively around the armature, the difference in the numbers of the

* See p. 126, on reactance voltage of the short-circuited coil.

spirals successively connected to the commutator segments is called the *number pitch*, or simply *the pitch*. In Fig. 50 the number pitch of the spiral is 1, and in Fig. 51 the number pitch of the coils is 1.

Commutator Pitch.—If the connections to the commutator be numbered successively, then, since there are always as many commutator sections as coils, these numbers may also be considered as applying to the coils. But the order of the ends of the armature coils may not be the same as the order of these connections; nor need they be connected to neighbouring segments. For this reason the term *commutator pitch* is used to denote the numerical difference between the number given to one commutator segment and that given to the segment to which the other end of the same armature coil is joined. In the ring windings of Figs. 50 and 51, and generally in windings of that type, the two ends of any coil are connected to neighbouring sections, so that the commutator pitch is unity.

Drum Windings.—From the preceding summary, it is clear that each turn or spiral of a ring armature consists of an active part lying on the outside of the core, which we shall hereafter refer to as the "*surface conductor*," and an inactive part forming the connection between one surface conductor and the next, of which a great portion lies inside the core. The essential distinction between the ring and the drum winding consists in the fact that the whole of the spiral is in the latter type placed on the outside of the armature, so that the portion which was within the core now forms a second active part or surface conductor, while the connections between the two project from the core ends, as shown in Figs. 3 and 71. Following out this idea, the author has in another place * shown how from the ring, the drum winding may be developed and understood. Here it is sufficient to indicate the line of reasoning by calling attention to two important points. First, since in the ring there is an inner wire corresponding to each surface conductor, the number of conductors on the surface of the corresponding drum will be twice what it was in the original ring. Consequently, whether the ring were wound with an odd or even number of spirals, the number of conductors on the drum becomes even.

Secondly, in the case of any spiral on the ring, the fact that the inner wire passes back through the armature causes the current to be apparently opposite in direction to that carried by the conductor upon the surface. Therefore, if the inner wire of the ring is to be placed upon the outside of the armature, to become useful, and if it is to be directly connected to the original surface conductor, it must lie in a field of opposite polarity to that in which the original conductor lay.

* "Armature Windings of the Closed-circuit Type."

In Fig. 52 we have a bipolar drum winding of ten turns composed in this way. The Roman numbers indicate the turns: for instance, I is the "old" conductor, and I′ the "new" conductor. Starting at I, the winding passes across to I′ and thence to II, and after that to II′, and so on. The connections to the commutator are at the junction of I and I′, II and II′, etc. If the conductors be numbered successively, as indicated by the Arabic numerals, we note that the "old" conductors are odd and the "new" even. The difference in the numbers of I and I′ is *odd*, and constitutes what is termed the front or *forward* * *pitch* (y_f). The difference in the numbers of I′ and II is also *odd*, and is called the *backward pitch* (y_b). The forward and backward pitches are 11 and 9 respectively in this case.

It is seen that for this simple drum winding $y_f - y_b = 2$, and obviously the total number of surface conductors is even. Since the

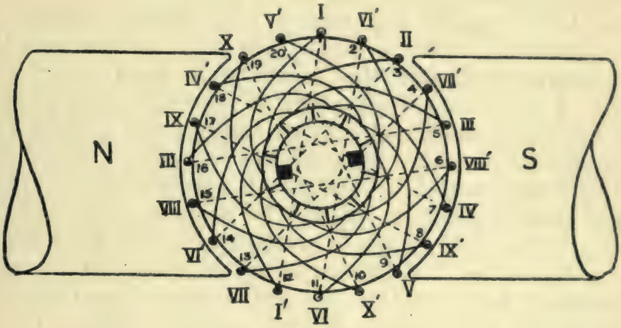


FIG. 52.

number of segments of the commutator is equal to the number of coils, each of which in this case consists of one turn, *the number of segments of this simple drum winding is half the number of surface conductors.*

The commutator pitch is unity. The brushes are placed on the segments connected to the conductors lying in the neutral position (*i.e.* lying between the poles). The formula for the armature resistance is exactly the same as for the ring armature.

Progressive and Retrogressive Windings.—In the preceding example, had the backward pitch been greater than the forward pitch by 2 (*i.e.* $y_b - y_f = 2$), then the winding would have been *retrogressive,* but the scheme is practically the same as the winding shown, which is said to be *progressive.* There is this difference, however, that the E.M.F. of the brushes is reversed, and therefore, in the case

* Note that the forward pitch throughout this work is the number-pitch of the conductors at the end remote from the commutator.

of a motor, the armature would rotate in the opposite direction for the same polarity of the fields. A bipolar progressive winding, then, is one in which the pitch at the end remote from the commutator is greater than that at the commutator end, the former being reckoned in the clockwise direction when the armature is viewed from the commutator end.

Multipolar Drum Windings.—Drum windings for armatures of multipolar machines are divided into two general classes—

(*a*) Multiple-circuit or lap-wound.

(*b*) Two-circuit or wave-wound.

The distinction between these classes lies in the fact that in the former the turns "lap back," while in the latter the coils progress from pole to pole. This will be seen more clearly from the following diagrams.

(*a*) *The simple multiple-circuit or lap winding.*

It has been pointed out that in the bipolar drum winding, a surface conductor under a north pole must be connected to one under a south pole, and the same holds good for the multipolar case. From an electrical point of view the poles need not be adjacent, but for geometrical reasons, because the system will then require a minimum length of wire, the conductors under adjacent poles are connected together. As far as possible, those lying under similar portions of the poles should be connected, as this tends to yield the greatest E.M.F. for a given flux. The total number of conductors divided by the number of poles is termed the *pole-pitch* of the winding; for it is the distance, expressed in terms of conductors, between the centres of two adjacent poles. If the forward pitch of the conductors be very much less or greater than the pole-pitch, both the conductors of a turn will come under the influence of one polarity and give zero E.M.F. for that turn. Nevertheless, windings with a pitch less than the pole-pitch have been suggested for the reasons referred to under the heading "Chord Winding" below. The forward pitch y_f is usually taken equal or slightly less than the pole-pitch, and must be *odd*. The backward pitch is for a progressive winding $= y_f - 2$, and for a retrogressive winding $= y_f + 2$, and must necessarily be also *odd*.

Moreover, if the direction of the E.M.F.'s in the conductors of these drum armatures be compared with those of the ring armatures (as indicated by the arrow-heads in Fig. 51), it will be found that the winding is naturally divided into as many circuits as there are poles, and that in any two neighbouring circuits the E.M.F.'s (and consequently the currents) must oppose one another. Therefore for effective collection of these currents brushes must be placed upon the commutator bars connected to the neutral conductors in which these currents would naturally meet. This results in as

many rows of brushes on the commutator as the machine has field-poles, alternate rows being connected together to form one pole of the machine.

For example, in Fig. 53 the number of surface conductors is 24, and of poles 4; hence the pole-pitch of the windings = 6.

Choosing an odd number for the forward pitch—

we have $y_f = 7$

so that $y_b = 7 - 2 = 5$ for a progressive winding.

In the figure, one set of connections, if carefully followed, will show the character of the simple multiple circuit or "lap" winding and its progression. The position and the number of the rows of

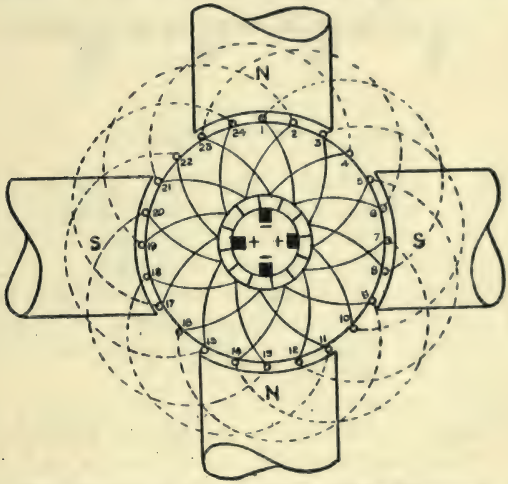


FIG. 53.—SIMPLE MULTIPOLAR DRUM WINDING.

brushes are both clearly indicated; there are 12 commutator bars, and the commutator pitch is 1.

Conventional Winding Table.—It is convenient to express the connections in a conventional winding table. This, for the winding we have just discussed, is as follows:—

1‾8_**3**‾10_**5**‾12_**7**‾14 **9**‾16, etc.

The paths of the currents through the armature for the particular brush position shown in the figure will be—

+ Terminal of machine	+ Brush 1	5, 12, 7, 14 8, 1, 6, 23	− Brush 2	− Terminal of machine
	+ Brush 3	17, 24, 19, 2 20, 13, 18, 11	− Brush 4	

Development.—In Fig. 54 the winding is supposed to be cut through at one point, removed from its core, and laid open on a flat

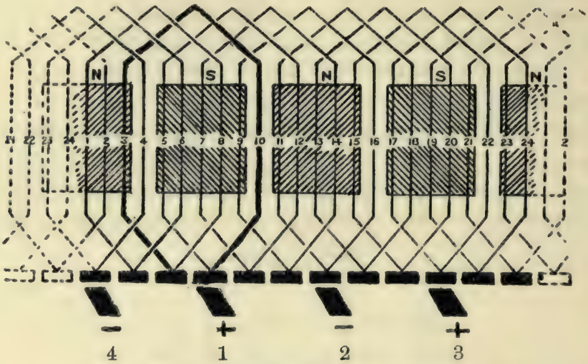


FIG. 54.—DEVELOPED LAP-WINDING.

surface. This is termed the "developed winding" or "development," and indicates clearly the characteristic lapping back, besides showing the position of the brushes. The characteristic of the winding is emphasized by the thickened coil or turn.

Chord Winding.—If, instead of taking a forward pitch, y_f about equal to the pole pitch, we choose a smaller pitch (still giving it an *odd* value), we shall obtain the narrow armature coil or so-called "chord" winding of Swinburne.

The advantage of the "chord" winding is that the currents flowing in the armature conductors in the neutral region are in opposite directions, so that their magnetic effect is zero, and therefore they play no part in armature reaction. On the other hand, some differential action tending to reduce the terminal E.M.F. often results (p. 94).

Resistance of Drum Winding.

If l' = total length of wire in the armature winding in inches,
a = cross-section of the conductor in sq. inches,
ρ = specific resistance of copper at specified temperature,
p = number of poles,

$$\left.\begin{array}{l}\text{Then resistance of armature circuit of}\\ \quad\text{a multipolar machine}\end{array}\right\} = \frac{\rho \cdot \frac{l'}{a}}{p}$$

$$\left.\begin{array}{l}\therefore\ \text{resistance of the simplex multiple}\\ \quad\text{circuit armature}\end{array}\right\} = \frac{1}{p} \cdot \frac{\rho \frac{l'}{a}}{p}$$

$$= \frac{\rho l'}{p^2 a} \text{ ohms}$$

Duplex Multiple-circuit or Lap Windings.—The forms of winding just described can be made "duplex," "triplex," etc., by placing a second or third winding upon the armature connected to commutator segments, which are regularly interleaved with those of the first winding; the various windings are then placed in parallel at the commutator, by brushes wide enough to touch at least as many segments as there are windings upon the armature. Thus, Fig. 55 shows a ring armature upon which are two separate and distinct windings, one shown full, A, B, C, etc., and the other dotted, marked A′, B′, C′, etc. It will be noticed that when the second winding is connected down to the commutator in the same way as the first

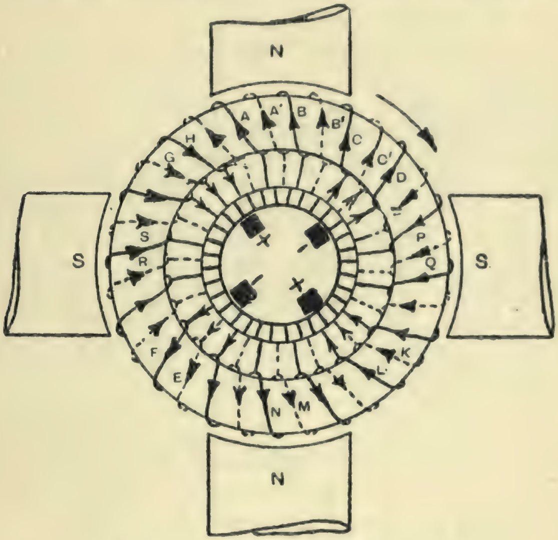


Fig. 55.—Duplex Ring Winding.

winding, its segments naturally occur alternately interspaced with those of the first winding, hence the commutator belonging to this form will have twice as many segments as the corresponding armature with a single winding.

Now, if each brush bearing upon the commutator be made so wide that it never touches less than two sections, we find that we have, with respect to these brushes, the two windings on the armature continually in parallel.

Object of a Double Winding.—The sole object of a double or duplex winding is to enable the armature to carry a large current, and to split up this heavy current into sufficiently small parts while it is being commuted.

A double winding, then, in general only supplies a method whereby we are enabled to change the output of a machine by simply altering the pitch of the winding, without changing either the number of turns upon the armature or the number of commutator sections.

Multiplex Multiple-circuit Windings (Multiple Re-entrancy): Notation.—In Fig. 55, instead of two windings in parallel, three, four, or any desired number for which there was room might have been wound. In the illustration, with only two independent windings the armature is said to be duplex doubly re-entrant; with three windings it is said to be triplex triply re-entrant,

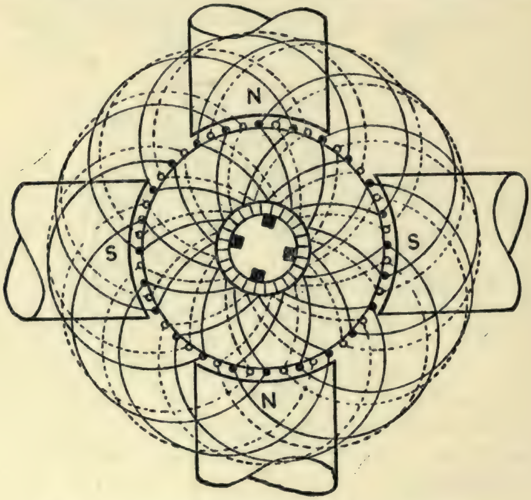


FIG. 56.—DOUBLY RE-ENTRANT LAP WINDING.

and so on. The following is the notation introduced to express this:—

Simplex winding,	o	
Duplex doubly re-entrant,	o o	(= 2 independent windings)
Triplex triply re-entrant,	o o o	(= 3 independent windings)

and so on for any number of independent windings.

Multiplex Drum Windings of Multiple Re-entrancy.—Now, as from the simplex ring winding the simplex drum winding was developed, so from the multiplex forms of the ring can multiplex drum windings be formed.

In Fig. 56 is shown a 4-pole double-wound or duplex drum winding. The two windings on the drum are entirely separate, and

the adjacent commutator segments belong respectively to the two windings. The black dots represent the conductors of one winding, and the circles those of the other. Suppose the dots to be numbered 1, 2, 3, 4, etc., and the circles 1′, 2′, 3′, 4′, 5′, etc. Each winding has 24 conductors, so that the pitches for *each winding* are $y_f = w/p$ approximately, where $w =$ No. of conductors in each winding.

$$\text{Since } y_f = 7, \text{ an odd number}$$
$$\text{and } y_b = 5 \text{ (for progressive winding)}$$

The scheme for the windings would be—

$$\mathbf{1}\underline{\ \ }8\overline{\ \ }\mathbf{3}\underline{\ \ }10\overline{\ \ }\mathbf{5}\underline{\ \ }12\overline{\ \ }\mathbf{7}\underline{\ \ }14\overline{\ \ }\mathbf{9}, \text{ etc.}$$

and

$$\mathbf{1'}\underline{\ \ }8'\overline{\ \ }\mathbf{3'}\underline{\ \ }10'\overline{\ \ }\mathbf{5'}\underline{\ \ }12'\overline{\ \ }\mathbf{7'}\underline{\ \ }14'\overline{\ \ }\mathbf{9'}, \text{ etc.}$$

The connections belonging to one winding are drawn with full lines, and those of the second winding are drawn dotted. The windings are put in *parallel* with each other by means of the brushes, each brush bearing upon not less than two segments of the commutator.

Pitch of Multiplex Lap Windings of Multiple Re-entrancy.—Since the number of conductors in each winding is even, the total number of conductors on the armature is divisible by 4, *i.e.* 2 × number of windings. Had there been m windings, the total number of conductors must, if each winding is independent and entirely separate from the others, be divisible by $2m$. Thus a winding must, in the first place, to be multiply re-entrant, have its conductors divisible by $2m$.

If the conductors be numbered consecutively so that No. 24′ becomes No. 48, we see that—

$$y_f = 14$$
$$\text{and } y_b = 10$$

Since y_f should be approximately equal to the pole-pitch, this value can be obtained thus—

$$\text{Pole-pitch} = \tfrac{48}{4} = 12$$
$$\therefore y_f = 12 + 2 = 14$$

The difference $y_f - y_b$ is now 4; that is, it is equal to 2 × number of windings. Had three windings been wound triply re-entrant, $y_f - y_b$ would have been 6, and generally for m windings m times re-entrant—

$$y_f - y_b = 2m$$

The winding scheme for the duplex winding now runs—

$$\mathbf{1}\overline{\ \ }15\underline{\ \ }\mathbf{5}\overline{\ \ }19\underline{\ \ }, \text{ etc. . . . 1st winding}$$
$$\mathbf{3}—17\underline{\ \ }\mathbf{7}\overline{\ \ }21 \text{ . . . 2nd winding}$$

For a 4-pole machine with 228 conductors—

$$y_f = 57 \text{ and } y_b = 53.$$

The winding table will run—

1¯58_**5**¯62_**9**¯66, etc. . . . 1st winding
3¯60_**7**¯64_**11**¯68 . . . 2nd winding

The commutator pitch y_c for the duplex winding is evidently 2, and generally for multiplex lap windings $y_c = m$.

Circuits through the Winding.—The diagrams show that the current divides at each brush into $2m$ paths, where m = number of windings, so that the resistance of the armature becomes $1/m$ times that of a simplex winding (p. 96).

Duplex Winding Singly Re-entrant.—In all the cases so far discussed the re-entrancy has been equal to the number of windings;

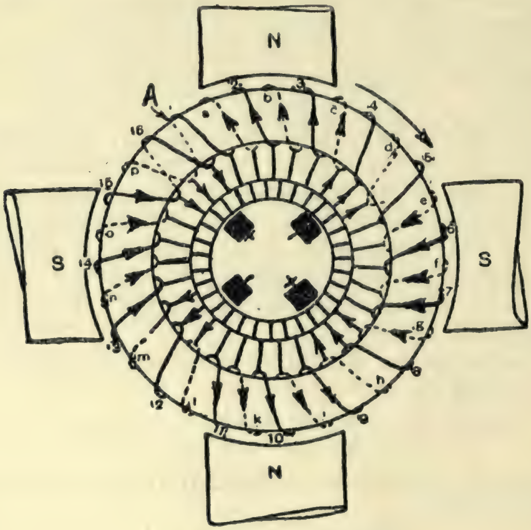


Fig. 57.

there is, however, another class of winding, certainly of little importance, but still sometimes to be met with, in which the re-entrancy is not equal to the number of windings, *i.e.* in which the various windings are not independent. Two examples will suffice to explain the general principle.

It is possible, in the case of a ring armature, to set out the spirals with such a pitch that the winding naturally continues twice or more times around the armature still finally closing on itself. This is exemplified in Fig. 57, which has 31 spirals upon it, with a

regular distance-pitch $\frac{2}{31}$ of the circumference. It is naturally a duplex winding, in which the two component parts are independent except at one point. In the figure one winding is dotted, the other in full line, and they meet at the beginning of the spiral 1 at the point A. When the brushes are wide enough to touch two sections at least, it is clear that the arrangement is simply another form of Fig. 55, but singly re-entrant because it has an odd number of spirals.

It thus appears that whether a ring winding is simplex, duplex, or triplex depends simply upon the *pitch chosen* with respect to the *number of spirals*, but the *re-entrancy* is dependent upon the *total number of spirals*. From which the following rule may be developed :—

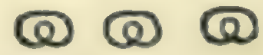
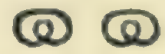
In a ring armature the G.C.F. of the number of windings and the number of spirals gives the re-entrancy.

And for the word "winding" we need the definition—

A set of spirals placed upon the core permanently in series with one another, and forming one complete annular helix, whether closing upon itself or not.

With this definition the series of the spirals marked 1 to 16 in Fig. 57 is just as much a winding as the series dotted in Fig. 55, although in the former case the particular set of spirals does not close upon itself.

Notation to express these Windings.—Just as a winding made up of two or more independent windings is denoted by an equivalent number of independent circles, so one that is double but singly re-entrant is denoted by a figure made up of two circles joined with a loop. The notation, therefore, expresses exactly the form in which the winding would appear if it could be removed from the core without cutting. Thus—

Simplex singly	re-entrant	is denoted by	o
Duplex doubly	„	„	o o
Triplex triply	„	„	o o o
Duplex singly	„	„	
Triplex singly	„	„	
Sextuplex triply	„	„	
Quadruplex doubly	„	„	

and so on.

Quadruple Winding Doubly Re-entrant.—Now, in Fig. 57 we might (if there were room in the diagram) insert another winding exactly like that already drawn, with the spirals of the new winding occurring alternately with those of the old. This would give 62 spirals upon the armature (in place of 31), a distance pitch of $\frac{4}{62}$ (instead of $\frac{2}{31}$); and on the armature there would be two windings, each double, and each independent of the other. This additional winding would again double the number of commutator sections, and

if the brush were wide enough, would simply be placed in parallel with the original winding. This arrangement would be described as a *quadruple multiple-circuit ring winding doubly re-entrant,* and it would be denoted by as explained above.

From these ring windings the corresponding drum windings, as described below, may easily be developed.

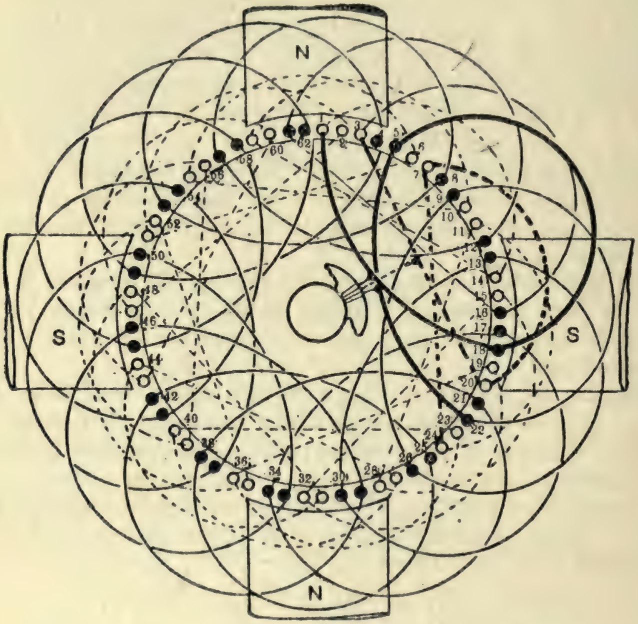


FIG. 58.—DUPLEX SINGLY RE-ENTRANT DRUM WINDING.

The Duplex Drum Singly Re-entrant.—The characteristic feature of the double or duplex drum winding was (see p. 99) the difference $y_f - y_b = \pm 4$, and the winding was doubly re-entrant provided the total number of conductors was a multiple of 4, or the total number of coils (of one turn each) was divisible by 2 (*i.e.* by the number of windings). If we take a drum to correspond with the ring of Fig. 57, there will be upon it 62 conductors (or 31 coils), and since the pitch is to be that of a double winding, we have—

$$\text{Pole pitch} = \tfrac{62}{4} = 15\tfrac{1}{2}$$
$$y_f = \text{say } 13$$
$$y_f - y_b = \pm 4$$
$$y_b = 17 \text{ (retrogressive)}$$

Thus the winding table is—

$$1\_18^{-}\mathbf{5}\_22^{-}\mathbf{9}, \text{ etc.}$$

But the winding is no longer doubly re-entrant, because 31 is no longer divisible by $m = 2$. The winding, as a matter of fact, *re-enters* itself after taking in *all* the conductors, thereby becoming *singly re-entrant*, exactly as is the case with the ring armature of Fig. 55. Following out the table, after the fifteenth lap, conductor 61 is reached, and we pass to No. 3 according to the table—

$$61\_16^{-}\mathbf{3}\_20^{-}7, \text{ etc.}$$

so that the winding naturally closes after fifteen more laps.

If each of the brushes covers, as before, *at least* 2 segments, the winding

$$1\_18^{-}\mathbf{5}\_22^{-}\mathbf{9}, \text{ etc.}$$

is placed in parallel with the winding

$$\mathbf{3}\_20^{-}\mathbf{7}\_24^{-}\mathbf{11}, \text{ etc.}$$

The number of armature circuits is, of course, the same as in the duplex doubly re-entrant winding, and the commutator pitch is likewise 2.

As far as commutation and current capacity of the armature are concerned, there is no difference between the two kinds of duplex windings. The question as to which would be used depends on the number of coils it is possible to place on the armature, whether even or odd.

Re-entrancy of Lap Windings.—It will be noted that the G.C.F. of the number of coils (32) and the number of windings (2) in the doubly re-entrant winding is 2, but that for the duplex singly re-entrant from it is 1. Whence we get a rule which, extended for multiplex lap windings, is stated thus—

The re-entrancy of multiple-circuit windings is the G.C.F. of the number of windings and of the number of coils.

From the considerations here outlined, a series of rules governing multiple-circuit or lap-windings can be drawn up.

Such a series is appended below, and refers to armatures with one turn per commutator section. For coil-wound armatures see p. 110.

Summary of Rules for Multiple-circuit Drum Windings.— 1. *Multiple-circuit drum windings* are necessarily *lap windings*, and follow the same general rules as ordinary ring windings, being simplex, duplex, triplex, etc., according to the number of pairs of parallel paths per pole in the armature.

2. A duplex or multiplex winding may be singly or multi-re-entrant, but this re-entrancy depends entirely upon the number of surface conductors.

3. There must be *in all cases* an even number of conductors, which

must be a multiple of the number of commutator sections, and in the case of slotted armatures a multiple of the number of slots.

4. The number-pitches for any single re-entrant winding must be odd and consequently for each independent winding or set of windings the number-pitches *considered without reference to other windings upon the same armature* must be odd. From this it follows that for a double-wound doubly re-entrant armature the number-pitches *may* be both even numbers; but generally odd numbers are taken.

5. The number-pitches for any number of conductors must differ by a number $= 2m$, where m is the number of windings.

6. The re-entrancy is = G.C.F. of $w/2$ and m, where w is the number of conductors.

7. The forward number-pitch will usually be approximately equal to w/p, where p is the number of poles.

8. The number of sets of brushes is equal to the number of poles, and each brush must cover a number of segments on the commutator greater than $m - 1$.

Equalizing Rings.—In multiple-circuit windings, it is essential that the E.M.F.'s in the armature sections be exactly equal, otherwise cross-currents will flow between them which increase the heating of the armature and thereby limit the capacity of the machine for a definite temperature rise. Since the number of conductors per section is equal, such difference in the E.M.F.'s of the sections can only depend on differences in the flux per pole. But it is conceivable with lap windings that differences in the depth of air-gap (as, for instance, when the babbit of the bearings wears away and the armature is out of centre) and differences in the reluctance of the various magnetic circuits might cause the flux to be unsymmetrically distributed between the poles. To prevent them from passing through all the armature-bars, these cross-currents are allowed to circulate through what are termed *equalizing rings,* whereby the potentials of the sections are levelled.

Suppose, for example, an armature with 144 conductors intended for an 8-pole machine. The number of conductors per pole = 18, and hence the number pitch between conductor 1 and the conductor similarly situated at an adjacent similar pole = 36. Hence conductors 1, 37, 73, and 109 occupy the same positions with respect to their north poles, and should therefore have the same potential if the fluxes cut were the same. In order to level their potentials, they are connected to ring No. 1. If other conductors are to be connected in the same way, we may fix on 9 equalizing rings. Then for the 36 conductors for each pair of poles we must space the connections to the equalizing rings every fourth conductor. Thus 5, 41, 77, 113 are connected to the next ring, and 9, 45, 81, 117 to the third, and so on. The more equalising rings used the less are the cross-currents in the

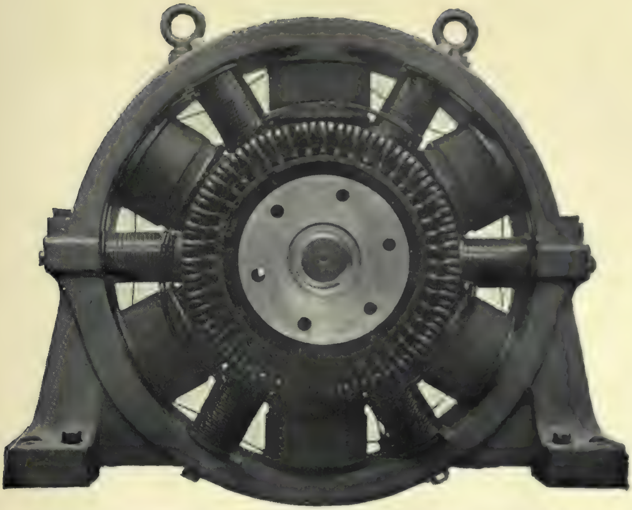


FIG. 59.—INTERPOLE MACHINE SHOWING EQUALIZER RINGS (PHŒNIX DYNAMO COMPANY).

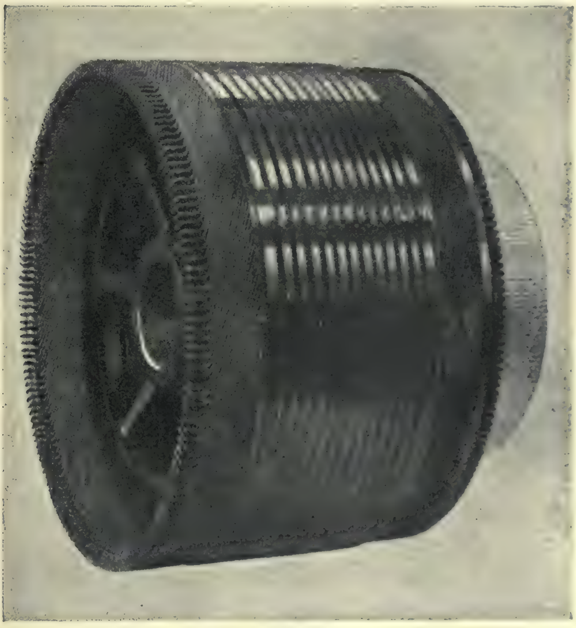


FIG. 71.—WAVE-WOUND ARMATURE.

[*To face p.* 104.

armature bars. The cross-currents are confined mostly to the equalizing rings and their connections, and tend by armature re-action to diminish the differences in the fluxes cut by the armature sections. The equalizing rings are connected either to the segments of the commutator which are joined to the conductors mentioned above, or else the rings are connected at the rear end of the armature to the rear-end connections as in copper strap-wound armatures. Fig. 59 shows a modern machine with equalizing connections to its armature.

Two-circuit or Wave Windings.—In the multiple-circuit winding on p. 95, after proceeding with the odd front pitch to the

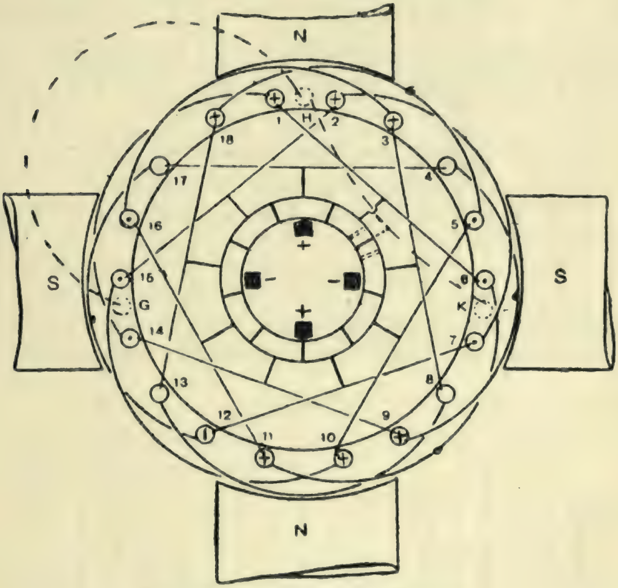


Fig. 60.—Simple Wave Winding.

adjacent pole, the winding returned to the next *old* conductor under the pole from which it started as shown by the thickened type in the winding table—

1	—	Even number	—	3	—	Even number	—	5
Old conductor		New conductor		Old		New		Old

If instead of returning to the next old conductor 3, which lies under the same north pole as 1, we proceed to an *old* conductor occupying a corresponding position under the *next* north pole, and thence to a "new" conductor under the next south pole and so on, we have the characteristic feature of the wave or two-circuit winding.

Fig. 60 is a diagram of a four-pole machine with 18 conductors on the armature. If a constant forward pitch of 5 were chosen, the scheme of winding would be—

1_6‾11_16‾**3**_8‾13_18‾**5**_10‾15_2
‾**7**_12‾17_4‾**9**_14‾1

Note that after visiting all the poles, namely, after the p^{th} connection ($p = 4$ in this case), we arrive at 3, the adjacent old conductor. Compare this with a lap winding, where after the *second* connection we arrive at this adjacent (old) conductor. In the two-circuit or wave winding, after another p $(= 4)$ connections the winding reaches 5, and so on; hence the winding progresses by what is termed *a creep* of 2.

Circuits and Brushes in Two-circuit Windings.—In Fig. 61 we have the development of Fig. 60; Fig. 62 is a second develop-

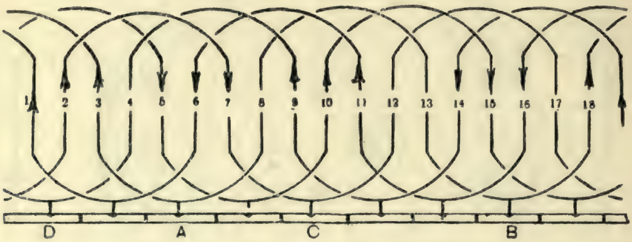


FIG. 61.

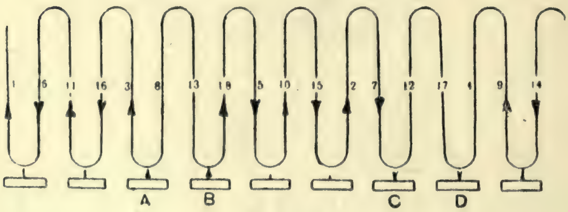


FIG. 62.

ment, giving the order of the conductors as they are connected (instead of the order of position). The electromotive forces induced under the respective poles are indicated by the direction of the arrows. Obviously brushes must be placed for collection, wherever in the winding two series of E.M.F.'s are in opposition. This is seen to be the case at C and at A. Two brushes at C and A divide the armature into *two circuits* only, whence the name. Starting with the positive brush, and following through the winding clockwise till

we reach the negative brush, it will be noted that one half the conductors on the armature have been passed over. The same procedure counter-clockwise will take in the other half of the conductors.

It is also clear from Fig. 62 that we might have obtained the same result by placing the brushes at B and D respectively instead of at A and C.

Now, from Fig. 61 it is seen that the segments A and C are 90° apart, and so also are the segments B and D. Either pair then may be used, and there is no reason why both should not be made use of; for the conductors lying between the two positive brushes C and D are inactive, and so also are those connected in series between the two negative brushes A and B. The addition of the two brushes B and D, then, practically amounts to an increase of the size of the brushes A and C, and affords an easy means of increasing the area of brush-contact without increasing the length of the commutator. For machines having p poles, a two-circuit winding would require positive and negative brushes on commutator segments spaced approximately $360/p$ degrees apart, and the number of paths through the armature winding is entirely independent both of the number of brushes and of the number of poles. This should be contrasted with the multiple-circuit winding, in which the number of paths through the armature is equal to the number of poles; whence the multiple-circuit armatures are sometimes termed *parallel wound*, as distinguished from two-circuit armatures, which are said to be *series wound*.

In conformity with the notation for the multiple-circuit armature on p. 95, the paths through this two-circuit winding with two brushes may be expressed thus:—

$$+\text{ brush C} \left\langle \begin{matrix} 7,\ 2,\ 15,\ 10,\ 5,\ 18,\ 13,\ 8 \\ 12, 17, 4, 9, 14, 1, 6, 11, 16, 3 \end{matrix} \right\rangle -\text{ brush A}$$

Resistance of a Two-circuit Winding.—This is easily seen to be given by the expression

$$\tfrac{1}{2}\frac{\rho l'}{2a} = \tfrac{1}{4}\frac{\rho l'}{a}$$

in which ρ = specific resistance of copper at the specified temperature
l' = total length of wire in the armature winding,
a = sectional area of the wire.

Formula for Two-circuit Winding.—The number of conductors of a two-circuit winding must be *even*. This is easily seen to be true, since the last connection must, after regular front and back pitches, naturally join to that end of the first conductor which is open. Evidently the number of end-connections (both forward and back) must equal the number of conductors, and must be even. The front

and back pitches (viz. end connections) are, wherever possible, taken the same, and must be *odd*, otherwise only half the conductors would be included.

$$\text{Thus } y_f = y_b = y \text{ (say)}$$

If y be made equal to the pole-pitch, then after p connections the winding closes, and only p conductors have been connected in ; so that for any large number of conductors we must take such a pitch as shall gradually include all the armature conductors. The pitch which fulfils this condition and is nearest to the pole pitch is given by the equation—

$$y = (\text{conductors} \pm 2) \text{ poles ; } i.e.$$
$$y = (w \pm 2)p$$

Creep.—The difference $py - w = 2$ constitutes the creep of the winding after p connections with a regular pitch of y. Starting at conductor 1 in any winding, after p connections we shall arrive at conductor 3, and after another p connections at 5, and so on till all the odd conductors are included. The even conductors must necessarily be also included, because the pitch is *odd*. On p. 105 we have a four-pole case with 18 conductors—

$$4y - 18 = 2$$
$$\text{hence } y = 5$$

Progressive and Retrogressive Wave Winding.—The formula

$$w = py - 2$$

will give a progressive wave winding, and

$$w = py + 2$$

a retrogressive wave winding.

Forward and Back Pitches.—In the general formula

$$w = py \pm 2$$

it has been said that y must be an odd number. If for given values of w and p, y comes out an even number, then odd forward and back pitches must be chosen.

$$\text{Thus for } w = 22 \text{ and } p = 4$$
$$22 = 4y - 2 \text{ (progressive)}$$
$$y = 6$$

Hence $y_f = 7$ and $y_b = 5$ are chosen, y being the mean of y_f and y_b.

In general, provided they are odd and give the correct mean pitch, any pair of pitches may be used ; and this is sometimes important to remember when a number of slots happens to be adopted which does not work in well with the best pair of pitches. Thus in the above case, if it were more convenient, $y_f = 9$, $y_b = 3$ might have been adopted, for the creep would have been the same; and for wave

windings the difference between the number-pitches does not decide whether the winding is simplex or multiplex, as it does for lap windings.

Commutator Pitch of Two-circuit Windings.—The commutator pitch is equal to the mean pitch, and in the above example is 6; for the commutator connections take place at every backward pitch, and the pitch from one backward connection to the next backward connection is equal to $y_f + y_b$. The number of commutator segments is, however, half the number of conductors; hence the commutator pitch is $\frac{y_f + y_b}{2}$, *i.e.* the mean pitch y.

Multiplex Wave Windings.—In Fig. 60 we could place on the armature another two-circuit winding as indicated by the dotted lines in the diagram. This second winding would be exactly similar to the first, and would have its commutator sections interleaved with those of the other. Such an arrangement would constitute a duplex wave winding doubly re-entrant. The formula for such a winding can easily be shown to be

$$w = py \pm 4$$

but y is now even, for y will be equal to $2y'$, where y' is the pitch for each winding considered separately.

For a *duplex singly re-entrant wave winding*, the formula is again the same, $w = py \pm 4$, but y becomes *odd*.

The general equation, then, for multiplex wave windings is $w = py \pm 2m$, where m = number of windings, and the general rules governing wave windings of one turn are collected together below. Coil windings are considered on p. 110.

Importance of Multiplex Two-circuit Drum Windings.—As the two-circuit simplex winding has only two circuits, the voltage developed is higher than that of the corresponding multiple-circuit case where there are more than two poles. Hence, if the current to be collected is too heavy for a two-circuit winding, it is usual to adopt the multiple-circuit type. Thus the multiplex forms of the two-circuit winding are relatively unimportant, and the multiplex forms of multiple-circuit windings are only necessary for such purposes as plating or for electric furnaces.

Rules governing Two-circuit Windings.—1. Two-circuit windings are of necessity "wave" windings, the system of end connections being such as to pass continually in one direction around the armature periphery when the winding is reduced to its simplest form—*i.e.* with one turn per commutator section.

2. These armatures may be simplex, duplex, triplex, etc., according to the number of parallel paths (through the armature) from brush to brush.

3. A multiplex winding may be singly or multi-re-entrant; this

re-entrancy depending upon the relationship between the pitch and the number of windings.

4. There must be in all cases an even number of conductors, which must be a multiple of the number of commutator sections (and also, in the case of a slotted armature, of the number of slots).

5. The rules for connection are given by the general formula

$$w = py \pm 2m$$

where w is the number of conductors;
p is the number of poles;
m is the number of windings required;
y is the mean pitch.

y must for all simplex windings be either an odd number, or two odd number-pitches must be used alternately as $(y + 1)$ and $(y - 1)$.

For multiplex windings, y should be either an odd number or "m" times an odd number, or some multiple less than m of an odd number. Attention to this will obviate the use of different pitches at the two ends of the armature.

The re-entrancy is given by the G.C.F. of m and y.

The number of sets of brushes absolutely essential is two, one positive and one negative. But there may be more sets in parallel with these according to the number of poles. Each brush must cover a number of segments on the commutator greater than $m - 1$.

Coil and Slot Windings.—All the armatures so far discussed, with the exception of Fig. 51, have been shown with but one turn per commutator section. This arrangement is only met with in large generators or motors; in smaller machines more than one turn per section is common, and the winding diagrams must be adapted to fit this case. Moreover, all modern armature cores, large and small, are slotted so that the winding may be better fixed and driven.

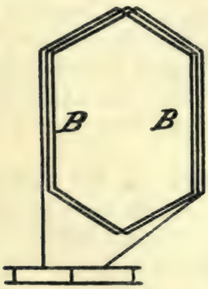


FIG. 63.—CONNECTION OF LAP-WOUND ARMATURE COIL.

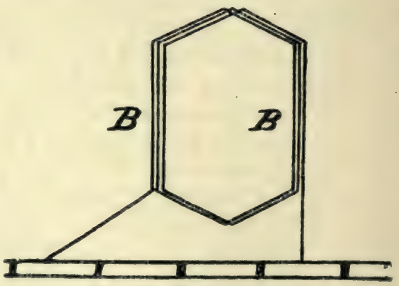


FIG. 64.—TWO-CIRCUIT ARMATURE COIL.

Coil Windings.—Now with reference to the first matter, *i.e.* the substitution of coils for single turns, consider one turn or element

of the development, Fig. 54. This is made up of two conductors, a back connection, and two leads to the commutator. Now substitute for this element the coil shown in Fig. 63, and it will be seen that the surface conductors become "coil-sides," and the connections to the commutator are the ends of the coil. Thus the pitch at the end remote from the commutator is settled by the coil shape and size, whilst that at the commutator end is settled by the connections to the commutator.

In an exactly similar way for two-circuit windings, one element or turn of the larger armature may in a smaller machine be replaced by one coil, as in Fig. 64, and here again the coil shape settles one pitch, the commutator connection the other.

Grouping of Conductors in Slots.—When slots are used, the surface conductors are naturally not distributed uniformly over the armature surface, but are grouped in the slots. So long as there is only one armature turn per commutator section, no trouble should be occasioned by the arrangement of the conductors in these groups, provided that the numbering of the conductors be carried out as suggested in Fig. 65, *i.e.* all even numbers be arranged at the top of the slots, all odd numbers at the bottom of the slots.

FIG. 65.—METHOD OF NUMBERING.

Fig. 66 illustrates well the method of counting surface conductors and pitches. It is a lap-wound 4-pole drum having 48 conductors and 24 slots.

$y_f = 15$, $y_b = 13$, so it is progressive and simplex, with a commutator pitch $= 1$.

In the same way, Fig. 67 illustrates a two-circuit six-pole armature with 64 conductors, and in the equation

$$w = py \pm 2$$
$$y = 11$$

The slots here number 32, and the commutator pitch is 11.

Meaning of Slot Winding-pitch.—When the coils or turns are grouped in slots in this manner, it is easiest for the winder, having numbered the slots consecutively, to reckon from the slot winding-pitch of the winding—*i.e.* the difference between the numbers given to the slots in which the two sides of a coil lie. If we be given this, and the commutator-pitch, and if all the turns be placed on the armature so that one side of each turn is at the top of a slot and the other side at the bottom (as in Figs. 66 and 67), the connection to the commutator is very easy and the result quite symmetrical. The slot winding-pitch is best reckoned at the end remote from the

commutator, *i.e.* in Fig. 66 it is 7, and in Fig. 67 it is 5. In general it is $\frac{1}{c}(y_f - 1)$, where c is the number of surface conductors grouped in one slot.

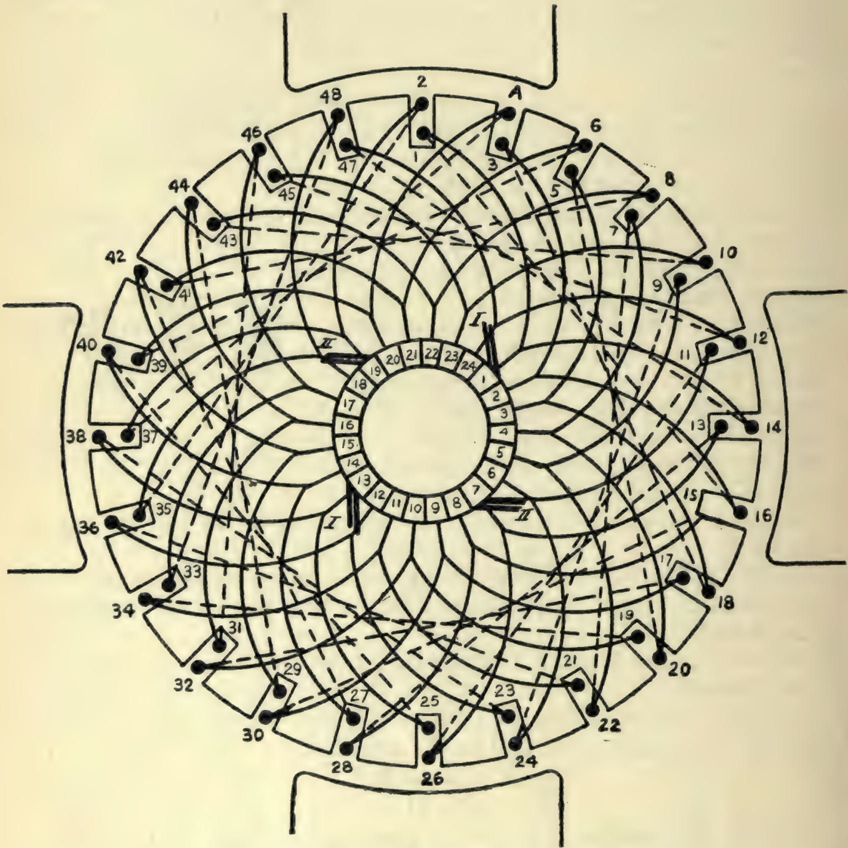


FIG. 66.—LAP-WOUND FOUR-POLE DRUM.

In Fig. 66 again the commutator pitch is 1, and in Fig. 67 it is $\frac{11 + 11}{2} = 11$. Given these instructions, it is easy for the winder to proceed by taking care that each turn spans a number of teeth equal to the slot winding-pitch, and that the commutator ends of these coils have the right commutator-pitch.

Grouping of Coils in Slots.—By following out the substitution

of coils for turns exactly as on p. 111, the meaning of the "slot winding-pitch" and "commutator-pitch" for such cases becomes apparent. The coil sides take the place of conductors in the slots, so that the coil itself must be made to span a number of teeth equal to the slot winding-pitch. The ends of the coil are brought down to the commutator, and there fixed to the segments with the correct

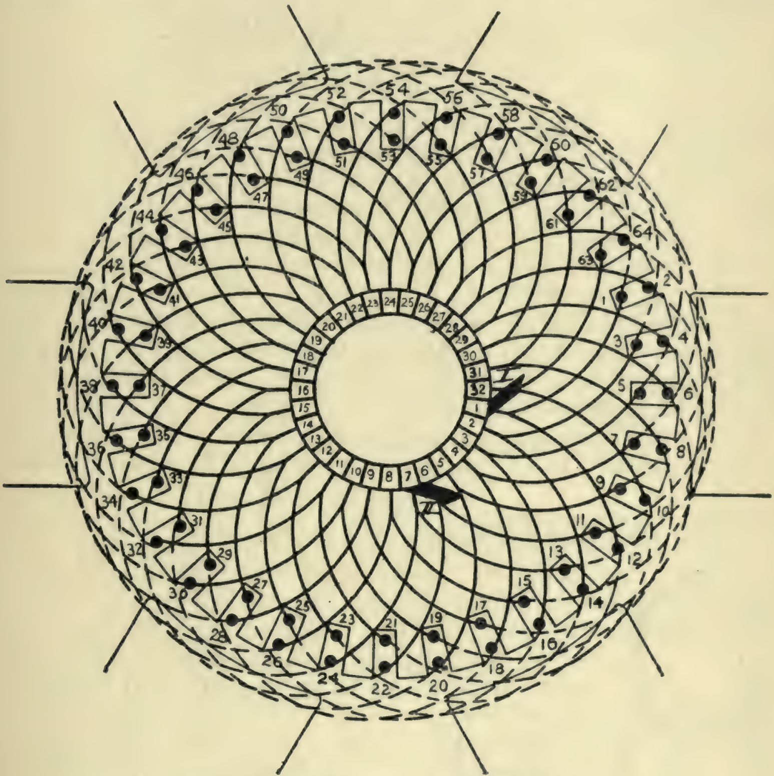


Fig. 67.—Wave-wound Six-pole Drum.

commutator pitch. For multiple-circuit winding this is easy enough, since the slot winding-pitch should be about equal to the pole-pitch, and the ends of the first coil, for simplex windings, must come to neighbouring segments (Fig. 63); while for the second coil one end will go to the right-hand segment of Fig. 63, say, and the other end will go to the next commutator segment to the right again, and so on.

As an example, suppose an armature for a 4-pole machine having 47 slots, 94 coils, each coil consisting of 4 turns. The commutator must have 94 segments, and each slot must hold 4 coil-sides. The

slot pitch may be $\frac{47}{4}$ = say 11, so that the first coil lies at the left-hand side of slots 1 and 12. The ends are brought down to segments 1 and 2. The second coil lies at the right-hand side of slots 1 and 12, and its ends, for progressive winding, go to segments 2 and 3, and so on.

In such a case if we count coil-sides as surface conductors, we have—

Total number of coil-sides 188 of four conductors each.

$$y_f = 45$$
$$y_b = 43$$

For wave windings the case is not quite so simple, because the arrangement must follow a rigid formula. The best way to proceed is to number the coil-sides as they will lie in their slots in accordance with Fig. 65. Then write the formula $w = py \pm 2$ in the form, coils = (pole pairs $\times$ y) $\pm$ 1. y is then the mean pitch, and from it y_f may be obtained as $= y$ if this be an odd number, or $= y \pm 1$, if y be even; y_b being similarly $= y$ or $y \mp 1$. The slot pitch is then obtained by dividing $(y_f - 1)$ by the coil-sides in one slot. Thus, suppose an armature having 41 slots, three coils per slot, *i.e.* six coil-sides in each slot, as in Fig. 87. If the field have 4 poles and the winding be two-circuit, we get—

$$\text{Total number of coils} = 123$$
$$123 = 2y \pm 1$$
$$\text{whence } y = 62 \text{ or } 61$$

This gives a choice of three windings, viz. —

$$(1)\ y_f = 61, \quad y_b = 61$$
$$(2)\ y_f = 61, \quad y_b = 63$$
$$(3)\ y_f = 63, \quad y_b = 61$$

Fig. 68.—Grouped Coils.

Since there are six coil-sides in one slot, the best value of y_f would be 61, for $(61 - 1)$ is divisible by 6.

Thus the slot winding-pitch becomes 10, the commutator-pitch 61. The diagram for the whole winding is shown more clearly in Figs. 68 and 129, from which it will be seen that each set of three coils is bound together so as to have the appearance of only one coil, although of course the ends are brought out separately, as depicted in Fig. 68.

These examples of the arrangement of coil windings in slots should, it is thought, suffice. Multiplex forms of such windings are of course rarely met with; for a multiplex winding is only of use after the winding has been reduced to one turn per segment and yet fewer bars in series are required.

Idle or Dummy Coils.—It is clear that though for lap windings any number of slots may be adopted, yet for wave windings the number of coils, and consequently of slots, is limited by the formula

$$\text{Coils} = (\text{pole-pairs})\ y \pm 1$$

If y is to be odd, clearly when the number of pole-pairs is odd the number of coils will be even, and *vice versâ*. Since the number of coils should be a multiple of the number of slots, the number of slots appropriate to an odd number of pole-pairs will be even, and *vice versâ*. But it sometimes happens that a standard armature is available, having a number of slots not according with the above statement. In such cases one or more coils may be "dummies," *i.e.* not connected to the commutator, but inserted to render the winding balanced and symmetrical.

In a similar way idle commutator-bars are sometimes used to make use of a standard commutator.

Best Numbers of Slots for Two-circuit Windings.—A little consideration shows that the series 29, 33, 37, 41, 45, 49, etc., is best for 4-pole machines with an odd number of coils per slot. Similarly, for 6-pole windings the series 26, 32, 38, etc., with two coils per slot gives good regular windings. And so on, for each number of poles there is for wave winding a preference in the number of slots to be used, so that when armatures may be required for either lap or wave windings the standard number of slots is usually chosen to suit the latter.

Comparison of Lap and Wave Windings.—1. *Appearance.* It is usually possible, with bar-wound armatures, to tell the style of winding from the appearance. For in the case of lap-windings the bar, as it lies on the armature, bends in the same direction at both ends (Fig. 69), while in the case of wave winding it bends in opposite directions, as is clearly seen in Figs. 70 and 71.

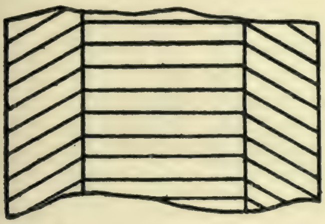


Fig. 69.—Appearance of Lap-winding.

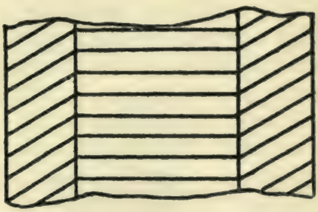


Fig. 70.—Appearance of Wave-winding.

2. *Electrical Balance.* It has been pointed out on p. 104 that lack of symmetry of the magnetic field causes parasitic currents in lap windings, and that equalizing rings are used to obviate these. With wave windings, since between brush and brush there are conductors

in series under all the poles, such an effect cannot follow from a lack of field symmetry, and equalizing rings are consequently unnecessary. On the other hand, with wave windings having an even number of pole-pairs an odd number of commutator sections is often a necessity; and this results in commutation which is not simultaneous at all the brushes, as is clearly evident from Fig. 60. Indeed, with all wave windings this must occur more or less, as by the formula the number of coils can never be an exact multiple of the number of poles. The result of this and of the system of connection is, that what is termed "selective commutation" occurs; that is, there is a tendency for the current to be collected at those brushes which have the lower resistance. Thus the author has known of cases where removing two rows (one + and one −) of brushes from a 4-pole wave-wound machine has distinctly improved commutation, although the current-density under the brushes must have been forced up very much thereby.

The Same Armature as Wave Wound and Lap Wound.—It is clear, from Figs. 63 and 64, that exactly the same coils may in some cases, be used for both classes of winding, the connections to the commutator only being changed. Thus a 4-pole wave-wound armature will give, in a certain field, at a certain speed, an electromotive force = V, and will carry without undue heating a current of C amperes.

If, on the other hand, we arrange it for a multiple-circuit winding, the coils will be the same, but the connections of the coils to the commutator will be changed. The armature will then give E/2 volts, and at the same current density as before 2C amperes, with the same field and at the same speed. This interchangeability is most useful to remember where many outputs are required from standard armatures; and in getting out new patterns of armature discs it should always be borne in mind that, while any even number of surface conductors may be used for a simple multiple-circuit winding, the same is not true for two-circuit armatures.

It is well, therefore, in designing the disc, to keep in view the possibility of having to use it for a two-circuit winding (*cf.* p. 192).

Finally, the differences between multiple and two-circuit armatures increase with the number of poles. Thus, if a 10-pole machine with a given number of surface conductors be arranged as multiple- and as two-circuit respectively, in the latter case it will give five times the voltage and one-fifth of the current that it did in the former, with the same number of lines per pole and at the same speed.

CHAPTER IX

COMMUTATION

Sometimes machines suitable for a certain output as far as temperature rise is concerned cannot be run at this output because of violent sparking at the commutator. Thus commutation plays a very important part in design.

Consideration of Commutation in Ring Armatures.—Consider a conductor such as (1), Fig. 72. As (1) moves round to position (5) it passes from a position in which it cuts maximum flux to a position in which it cuts no flux.

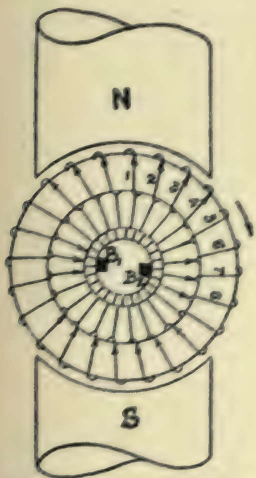


Fig. 72.—Commutation in a Ring Armature.

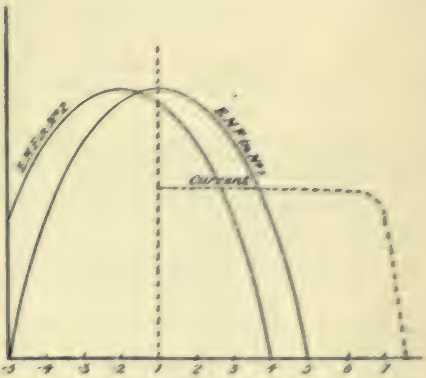


Fig. 73.—Curves of E.M.F. and Current per Conductor.

Thus plotting the E.M.F. generated in (1) for the various positions (1) to (5), we obtain a line like that shown in Fig. 73, graph (1), the E.M.F. falling from a maximum to zero.

Now consider conductor (2). The E.M.F. curve will be exactly similar to that of (1), but will precede it by a time proportional to the distance between the two conductors.

The armature E.M.F. thus forms a polyphase system, the number of phases being equal to the number of commutator segments; but though the E.M.F.'s at any given instant in turns (1), (2), and (3) thus differ, the current in the three turns is the same.

Now, it is to be noted that the turns at (7) and (8) in the figure are short-circuited under the brush, so that the current will fall to zero as (1) reaches (7). The current is then constant from positions (1) to (6), and falls to zero about position (7); it rises to its normal value again beyond position (8).

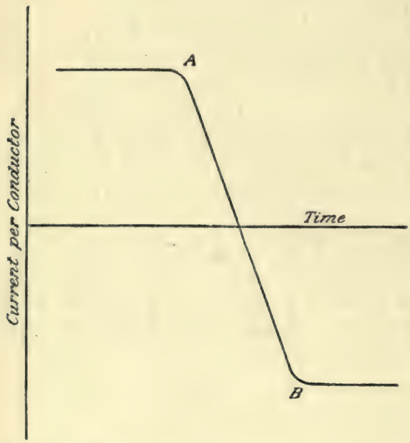


FIG. 74.—FALL OF CURRENT IN CONDUCTOR UNDERGOING COMMUTATION.

Thus the continual falling away and re-establishment of the E.M.F. cannot be taken advantage of to cause the current to change in a similar gradual manner; and it is the effect of collecting the current *at a particular point* that causes the difficulty in commutation, because the current in each turn must fall to zero, and then rise to its normal value in the opposite direction (Fig. 74) when the coil is connected to a segment under the brush.

From the number of coils undergoing this complete change per second is derived the *frequency of commutation*; obviously—

(1) The greater the width of brush the more time is there to effect the change.

(2) The greater the speed the less is this time.

Now—

$$\text{Time of commutation } (= t_c) = \frac{\text{brush width in segments } (= g)}{\text{peripheral comr. speed in segs. per sec.}}$$

A complete cycle is taken to be twice that represented between A and B (Fig. 74).

Let $\sim$ = complete cycles per second.

$$\text{Then } \sim \text{ of commutation} = \frac{\text{peripheral speed}}{2 \text{ (brush width)}} (= \sim_c)$$

Let n = revs. per second;

S = number of segments.

$$\text{Then } \sim_c = \frac{\text{S}n}{2 \text{ (brush width)}} = \frac{\text{S}n}{2g}$$

Commutation in Drum Armatures.—Though the case as hitherto presented refers particularly to ring armatures, it applies equally well to those that are drum-wound. It is merely necessary to substitute for one turn of the ring in Fig. 72 one turn of the drum Fig. 52, when it is clear that we have a case analogous in all respects to that of the ring. The only difference is the *position* of the loop or turn considered, which for the present is immaterial.

Methods of Commutation.—If a circuit carrying a current be suddenly opened, an arc or spark will almost invariably appear at the moment and place of opening, unless the circuit be very nearly non-inductive. It is possible, however, to reduce the current to zero in a circuit either (1) by introducing an appropriate counter E.M.F., or (2) by introducing a resistance gradually increasing in value up to infinity.

These considerations lead to two methods of satisfactorily accomplishing commutation, called respectively—

(1) *E.M.F. commutation* and (2) *Resistance commutation.*

In E.M.F. commutation the current in the coil under the brush is stopped and reversed by an opposing E.M.F. set up in the coil, as, on leaving the brush, it comes under the influence of the "leading" pole-tip* or special commutating pole.

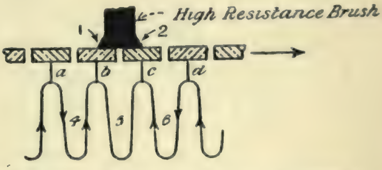


Fig. 75.—Resistance Commutation.

Resistance commutation is brought about as follows: Imagine the commutator segments moving as indicated by the arrow, Fig. 75. If, to begin with, the segment b just touches the tip (1) of the brush, the resistance between b and the brush will be very high, so that most of the current collected will flow through c. Conductor (5) will then have to carry nearly all the current in (4). As b comes further under (1) the resistance decreases so that more current flows through b, *i.e.* the current through (5) is decreasing. When the brush becomes symmetrically placed with regard to (4) and (5), as in the diagram, practically the same amount of current will flow through b as through c, half the total flowing through each. Thus the current in (5) has been reduced to zero. Further motion increases the resistance of c, so that some of the current will flow through (5) in the reverse direction and pass through b. Finally, the resistance of c is so great that all the current flows through (5) in the reverse direction to the original current.

Thus the current has been caused to change from a certain value through zero down to the same value in the opposite direction. So

* The "leading" pole-tip of any pair of poles in a dynamo is that pole-tip towards which the coils of the armature are moving as they leave the brush, *i.e.* as marked A in Fig. 76.

when the break between the brush and *c* actually occurs, there is very little current flowing from it to the brush, and therefore no spark. This is the foundation of so-called resistance commutation.

The two methods may be combined in practice, *i.e.* we have a third method possible, viz.—

(3) *Mixed E.M.F. and resistance commutation.*

(1) **E.M.F. Commutation.**—For E.M.F. commutation a strong and constant field must exist under the leading pole-tip of a dynamo (or trailing pole-tip of a motor).

Failing this, it is necessary for every increase in load to move the brushes forward until the coils under commutation are in a strong field, or to use special devices to effect commutation. The former must be avoided, not only on account of the constant attention the machine would need, but also because, in addition to good commutation, small regulation is required, and this means small movement of the brushes. As an ideal position the brushes should be on the no-load neutral line. In the case of reversible motors, such as tramway and railway motors, it is essential that the brushes should be on this line.

Of special devices, very many have been proposed, but only a few of these are satisfactory; the best are given below.

(1) *Special Commutation Coils on the Armature.*—These consist of a few turns of wire inserted on the armature in between the segment and main winding, and spaced so that when a segment is under the brush the commutating turns connected to it will be under the leading pole, and so in a strong field. This produces the necessary reversing E.M.F. The brushes may then be arranged for any position on the commutator. This system is known as *Sayers winding* (Fig. 76).

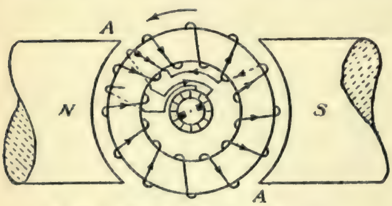


FIG. 76.—SAYERS WINDING.

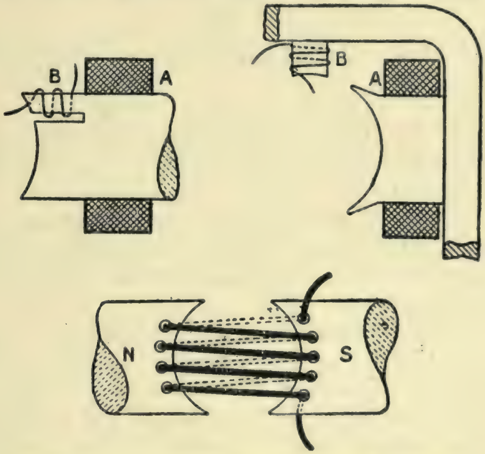
Sayers winding is satisfactory for small machines, but it is extremely sensitive, and, for some unknown reason, fails for machines above about 20 K.W. The principle itself is sound, as may be seen on small machines, where it is possible to obtain sparkless running even with a backward lead (for dynamos). The principle may be carried so far that the machine will run under the armature field alone, the main field coils bein disconnected. The winding is, however, rather costly.

(2) *Special Shapes of Pole-tips.*—The steady field under the pole-tip may be obtained by means of saturation in the pole-tips, for which various devices have been suggested. One method consists in cutting slots in the poles near to the leading pole-tip, and another plan is to specially shape the shoe so as to make it very narrow at the saturated tip.

(3) *Specially Wound Tips* may be used, the coil being placed in series with the armature (Fig. 77); from (3) has been deduced—

(4) *Special Commutating Poles.*—In this case, instead of winding the tip of the pole, the main pole is left unchanged, and a small pole introduced which carries the coil for producing the reversing field (Figs. 78 and 59).

(5) *Ryan Winding.*—Instead of having a separately wound coil to saturate the pole-tip and so decrease the effect of the cross ampere-turns, the latter may be neutralized by turns passing round the armature and through the poles (Fig. 79). For D.C. machines this winding has not been adopted except in cases of very high speed,



FIGS. 77, 78, AND 79.—AIDS TO COMMUTATION.

because of the large amount of copper required, which may amount to more than that necessary for the armature.

Usual Methods.—Of all the methods for improving E.M.F. commutation by special windings, only two are of any use in practice, namely, (1) the commutating pole or interpole, and (2) the Ryan winding; and since the latter is expensive, we are only left, for ordinary purposes, with the commutating pole.

Calculation of E.M.F. Commutation.—It can be shown that for perfect commutation, the commutating E.M.F. to be introduced into the coil is given by the expression *—

$$e = \mathrm{C}_w r \frac{1 + \varepsilon^{-\frac{r}{\mathrm{L}_c}t_c}}{1 - \varepsilon^{-\frac{r}{\mathrm{L}_c}t_c}}$$

* See Appendix V. and paragraph on effect of self-induction, *infra*.

where r = resistance of the circuit of the coil under commutation;
L_c = coefficient of self-induction of the coil;
t_c = time of commutation;
C_w = the current per conductor in the armature.

Of these quantities C_w is known, and t_c is easily calculated as shown on p. 118. The value to be taken for r is dependent upon many considerations; but with proper E.M.F. commutation it should be possible to use very low resistance brushes. In this case, then, the safest course is to regard r as simply the resistance of the coil.

Calculation of L_c.—We have for any coil—

$$L_c = \frac{(\text{flux set up in 1 turn by 1 amp. flowing therein}) \times \text{turns} \times \text{turns}}{10^8}$$

i.e. $L_c = \frac{T^2 f}{10^8}$, where T = No. of turns per coil, and f = field set up by 1 amp. in 1 turn.

Value of "f" by Hobart's Method.*—The average field set up by 1 amp. turn was tested for various slots, and the following conclusion was arrived at:—

For a ratio of slot-width to slot-depth not exceeding 3·5, *the flux set up is on the average* 10 *C.G.S. lines per inch of slot (axial length), and* 2 *C.G.S. lines per inch length of conductor not lying in the slot (for ordinary drum armatures).*

Now, for ordinary drum armatures, the length of mean turn on the armature = L.M.T.$_a$ = 2 × net iron length + 3 × pole-pitch. ∴ C.G.S. lines per amp. turn = 20 net length + 6 pole-pitch. Now, for all D.C. armatures there are 2 coil sides per slot simultaneously undergoing commutation under different brushes, so that the interlinking field will be produced by both these conductors or coil-sides in the one slot.

Thus—

$$\left.\begin{array}{l}\text{Lines per amp. turn, reckoning}\\ \quad\text{neighbouring coils}\end{array}\right\} = 40 \text{ net length} + 6 \text{ pole-pitch}$$

$$i.e.\ f = 40\,\text{l.} + 6\,P_p$$

This, however, is only true under the condition that the brush-width = 1 segment, *i.e.* No. of coils short circuited by brush = 1.

If the brush short circuits g coils, then—

$$f = (40\,\text{l.} + 6\,P_p)g$$

The coefficient of self-induction is thus reduced to terms of the armature dimensions.

It may be objected that the above calculation is too rough to be reliable. As a matter of fact, however, it gives results accurate

* See *Journ. Inst. E.E.*, vol. 31, p. 189.

enough for ordinary work. The value of f is that for coils in slots lying in the interpolar spaces, and when the interpole shoe is brought near to these it naturally increases the value of L_c. This increase depends on the dimensions of the shoe and on the length of the gap. It may be calculated either by means of Carter's formula (p. 47), or a conventional allowance may be made to which a fair approximation is—

$$f = (50 \text{ net length} + 6 \text{ pole-pitch})g$$

In any case the value of e thus calculated will be higher than is actually required, because the resistance of the brush, which

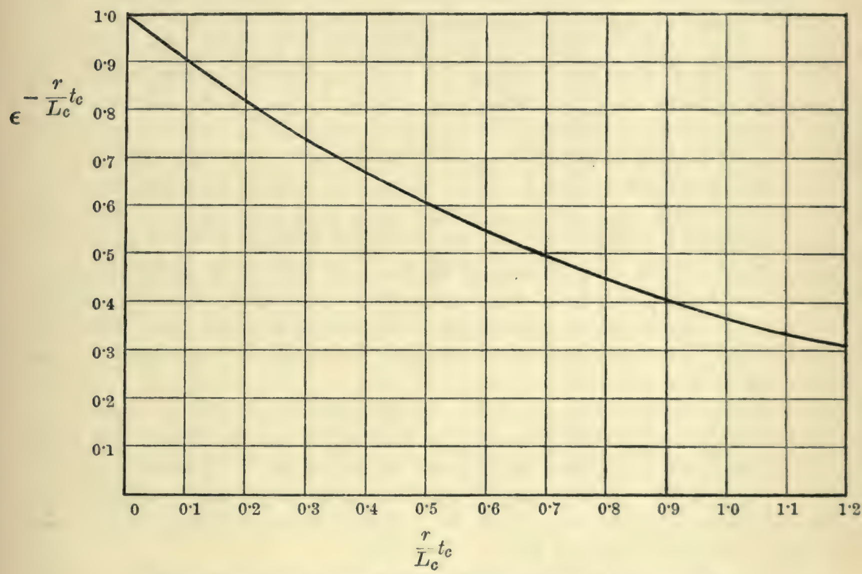


FIG. 80.—CURVE FOR E.M.F. COMMUTATION CALCULATION.

helps commutation, has been entirely neglected. It is better, however, to have the interpole ampere-turns a little large, as by a diverter rheostat they can be adjusted to suit exactly the working conditions.

To facilitate the calculation of e by the above methods, Fig. 80 is here given.

Interpole Flux.—From the equation—

$$e = 2 \times \text{turns per armature coil} \times \text{flux per interpole} \times 10^{-8} \div t_c$$

the value of the average flux per interpole can be obtained. The interpole air-gap may be taken the same as that under the main poles, and the density at the interpole shoe as 40,000 lines per

square inch. The ampere-turns required for this flux may then be calculated exactly as for the main magnetic circuit. The interpole arc may be taken wide enough to ensure the coil lying in the field all the time its segments are under the brush. The axial length of shoe is then fixed by the value of the flux in the pole.

Interpole Flux from Reactance-voltage.—Many writers calculate the interpole flux from the value V_r as estimated below. The author believes the method outlined above to be more accurate and quite as simple, though it gives higher values than are obtained usually from V_r.

Ampere-turns for the Interpole.—The densities in the whole of the interpole magnetic circuits should not be so high as to cause much saturation in the various iron parts, or else the interpole field will not be proportional to the interpole ampere-turns, *i.e.* to the armature load, as for proper commutation it should be. The only part where saturation is liable to occur is the pole core, which is usually a steel casting or iron forging, and in which the density should not exceed 100,000 lines per square inch. In order properly to calculate the pole area, it is necessary that the interpole leakage-factor should be carefully estimated. This is done exactly as for the main poles, with due attention to the fact that practically leakage will only take place from the interpole to the next pole of *opposite sign*. It is usual to calculate this leakage-factor for full-load conditions, and to assume it constant; its value is very large, often it is 1·8 (see p. 209).

Having thus obtained the ampere-turns necessary to carry the flux already calculated though the magnetic circuit, there must be added to them a number of ampere-turns equal to those existing at each pole of the armature; for the latter, as cross ampere-turns, act in opposition to the interpole, and must be neutralized before any interpole flux can be set up.

Thus the total ampere-turns for the interpole consist of the armature ampere-turns per pole + the ampere-turns necessary to set up the flux for the E.M.F. e. The total is usually not very different from 1·3 times the armature ampere-turns per pole. An example of the calculation of interpoles is given on pp. 195 and 209. In machines where the pole-tip flux is relied upon for commutation, it follows from the foregoing considerations that the field ampere-turns per pole must not be less than 1·3 times the armature ampere-turns per pole; but such machines are now practically obsolete.

Interpole Loss.—The watts expended in the interpoles depend to some extent upon the shunt field loss and the efficiency required. Usually like the loss in compounding coils, it ranges from 1 per cent. in small machines to ½ per cent. in large machines.

Resistance Commutation.—It has been shown that the current

in a coil may be caused to start from a certain maximum, decrease, pass through zero, and rise to a maximum in the opposite direction by using a high-resistance sliding contact.

By moving the brush uniformly along we ought to obtain a curve of current change similar to the one shown in Fig. 81, known as the *commutation curve.*

Effect of Self-induction.—Self-induction, however, which is present in all armatures, modifies the shape of this curve; and since the coils lie on an iron core, the resistance of any coil may, and usually is, much smaller than its self-induction.

Now, in the ordinary case, the current will fall, when the coil is

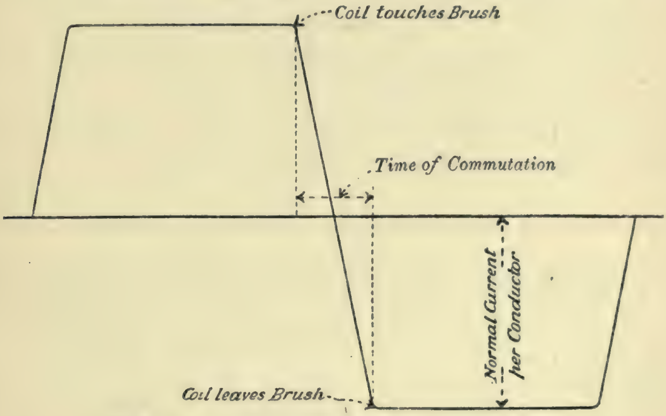


FIG. 81.—PROCESS OF LINEAR COMMUTATION.

closed by the brush, according to an exponential law. Thus

$$C_i = C\epsilon^{-\frac{r}{L_c}t}\,*$$

where r = resistance, and L_c = the coefficient of self-induction of the coil, and C is the value of the current in the coil just before it is closed by the brush. On plotting this curve, it will be found that the current never actually falls to zero; and also that the greater r is with respect to L_c, the smaller may be the time necessary for a given fall of current.

Similarly, the current in an inductive circuit takes a definite time to rise: its value at any instant being given by the equation—

$$C_i = \frac{V}{R}\left(1 - \epsilon^{-\frac{r}{L_c}t}\right)*$$

$\frac{V}{R}$ representing the maximum current in the coil.

* *Cf.* Appendix V.

Thus if the coil has a large inductance, the natural time taken to fall to zero and rise to a maximum again is correspondingly large.

If the natural time taken to fall and rise again is greater to any extent than the time the segment is under the brush, the current will tend to flow from the segment to the brush as an arc or spark. Hence it is essential for resistance commutation that the resistance of the coil-circuit be large and its self-induction small.

This leads to the high resistance brush, *i.e.* the carbon brush.

It also limits us to drum-armatures as the self-induction of a ring-winding is so much greater. In ring-winding there is a good magnetic path for any leakage flux set up by commutation, and the number of turns is the number of surface conductors; whereas in drum-winding half the path is through air and the number of turns is *half* the number of surface conductors. The following are the desiderata for successful resistance commutation :—

(1) At the instant at which the brush unites two segments the current which flows across the small section of brush-tip must not be large enough to cause the tip to get very hot or to glow.*

(2) The ratio $\frac{r}{L_c}$ for the short-circuit must be such as to give the current sufficient time to die away before the coil leaves the brush.

(3) At the instant when the coil is unshort-circuited, the current passing from the brush to the segment leaving the brush must be small enough to prevent heating or sparking at the brush-tip.*

In the above expressions r refers to the complete resistance of short-circuit, *i.e.* armature-coil, contact, and brush, of which the contact forms by far the greater part.

It is not only necessary to limit L_c, for commutation depends upon the voltage set up by the current, *i.e.* upon $L_c \times \sim_c \times C_w$.

Thus we must limit this product rather than any component; and of these factors L_c for a particular kind of winding and armature may be considered constant.

Assuming, then, L_c constant and neglecting minor factors, the curve of current change is represented very roughly by the graph shown (Fig. 74). By means of the oscillograph the actual current variation has been obtained for various cases, and has been found to be very irregular.

Reactance Voltage.—The value of e calculated above for E.M.F. commutation is a reactance voltage in the coil under commutation. Although it is, as the author thinks, a better guide to excellent commutation than that presently to be defined, yet it does not

* For an excellent discussion of the current-density at the brush during commutation, the reader is referred to the article by F. W. Carter in the *Elec. World* (N.Y.), March 31, 1910.

so easily lend itself to embodiment in the main design equations. Now, Hobart has suggested that, as the shape of the curve of current change is so irregular and uncertain, we may, for comparative purposes, assume it to be a half sine-wave, and calculate the reactance voltage from this. It has also been suggested, with more reason, that if we are to make crude assumptions the graph might be taken as a straight line.

Below, the reactance voltage is calculated for both these assumptions.

Let t.p.s. = turns per segment, *i.e.* turns per armature coil.

$$\text{Then } L_c = \frac{(40l \times 6P_p)g(\text{t.p.s.})^2}{10^8}$$

$$\text{Reactance } x = 2\pi{\sim_c}L_c$$

where $\sim_c$ = frequency of commutation

$$= \frac{Sn}{2g} \text{ (p. 118)}$$

$$\therefore \text{ reactance} = \pi\frac{Sn(40l + 6P_p) \times (\text{t.p.s.})^2}{10^8}$$

If C_w = current carried by any armature coil,

then reactance voltage = $C_w x$

and reactance voltage per commutator segment =

$$V_h = \frac{C_w \pi Sn(40l + 6P_p)(\text{t.p.s.})^2}{10^8}$$

where V_h = reactance voltage according to Hobart, *i.e.* assuming a sinusoidal change of current.

For a straight-line graph the reactance voltage is denoted by V_r.

$$V_r = V_h \times \frac{2}{\pi} = \frac{2C_w Sn(40l + 6P_p)(\text{t.p.s.})^2}{10^8}$$

Example.—Armature net length = 5 inches.
Length of mean turn = 40 inches.
Current reversed per section = 100 amps.
Frequency of commutation = 500.
Turns per segment = 1.
Width of brush = 3 segments, *i.e.* turns short circuited = 3.
Find the reactance voltage.
We have—

$$C_w x = \frac{\pi n S \times (40l + 6P_p)(\text{t.p.s.}) \times C_w}{10^8}$$

net length = 5″
L.M.T.$_a$ = 40″ ∴ free length = 30″
i.e. $P_p = \frac{30}{3} = 10''$

$$\text{Thus } V_h = \frac{\pi Sn \times (200 + 60)(1)^2 \times 100}{10^8}$$

$$\sim_c = 500 = \frac{Sn}{2g} \quad \therefore Sn = 500 \times 2g = 3000$$

$$i.e.\ V_h = \frac{\pi 3000 \times (260) \times 100}{10^8} = 2{\cdot}45$$

Output and Linear Reactance Voltage.—If in the formula for V_r we substitute the value $\frac{2C_w Sn(\text{t.p.s.})}{10^8} = \frac{EC}{\text{poles} \times \text{flux per pole}}$, which is derived directly from the E.M.F. equation, we get the form—

$$V_r = \frac{EC(40l + 6\text{p.p.})(\text{t.p.s.})^*}{\text{poles} \times \text{flux per pole}}$$

in which EC stands for the total watts generated in the armature. This equation is extremely useful, as will be seen later.

Machine Dimensions and Reactance Voltage.†—It has been already pointed out (p. 6) that a field-pole approximating to a circular or a square section will usually give economical results; or, at the very least, these shapes form a good starting-point for comparative estimates of different designs. From this assumption it is easy to show that with normal magnetic densities there is a distinct connection between size of pole and diameter of armature; which connection has been shown on p. 20 to be—

$$D = \frac{p \,.\, d}{1{\cdot}5\lambda} \text{ for round poles}$$

If for λ we substitute the mean value 1·15, and insert in the formula for V_r after some transformation, ‡ we obtain the equations—

$$V_r = \frac{86d \,.\, (\text{t.p.s.})^2 \,.\, Sn \,.\, C_w}{10^8} \text{ for circular poles}$$

$$\text{and } V_r = \frac{93d \,.\, (\text{t.p.s.})^2 \,.\, Sn \,.\, C_w}{10^8} \text{ for square poles}$$

where C_w is the current in any armature turn, d is the diameter or length of side of the pole, as the case may be.

Machine Output, Dimensions, and Reactance Voltage.—Going a step further, we may express the output in terms of the reactance voltage and flux (see Appendix VI.). We then get the forms—

A. For multiple-circuit windings—

(1) $V_r = 0{\cdot}55 \times$ K.W. per pole $\times$ t.p.s $\div$ pole diameter for circular poles.

(2) $V_r = 0{\cdot}465 \times$ K.W. per pole $\times$ t.p.s $\div$ pole side for square poles.

* "Flux per pole" in these formulæ is obviously the useful flux per pole, *i.e.* in the armature.

† See Appendix VI.

‡ See Appendix VI.

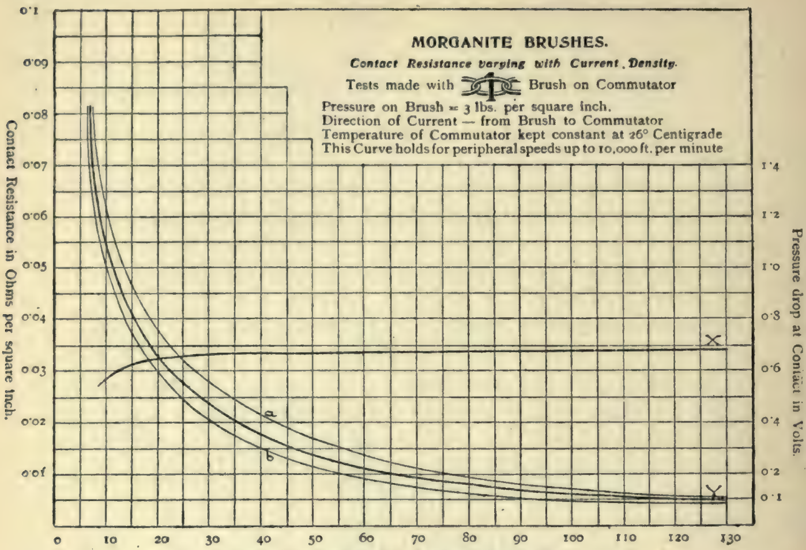
B. For two-circuit windings with two brush rows—

(1) $V_r = 0{\cdot}275$ K.W. $\times$ t.p.s. $\div$ pole diameter for round poles.
(2) $V_r = 0{\cdot}232$ K.W. $\times$ t.p.s. $\div$ length of pole sides for square poles.

These formulæ are of great assistance in determining the limiting dimensions for preliminary designs. They have, however, limits themselves. For reference to p. 122 will show that the value f was obtained on the assumption that all the g coils undergoing commutation lay in the same slot. This, with ordinary brush widths, is the case. But if the brush width be increased so that g is greater than the number of coils per slot, the reactance voltage per segment is not further increased. Thus V_r has its maximum value when $g =$ the coils per slot. And in those rare cases when g is greater than the number of coils per slot, the latter value is to be adopted in the calculation in place of g. We say "those rare cases," because they only occur when either the number of slots is abnormally large, or the width of brush so great as to endanger sparking, on account of the fact that some of the coils will be well under one pole-tip or the other, while their segments are still under the brush.

Limits of Linear Reactance Voltage.—Properties of Brushes.—The limits to be set to the reactance voltage depend upon the quality of the brush used. Senstius * has shown, and the author's experience confirms the view, that the reactance voltage which a brush of given quality will stand without sparking, is inversely as the current density at which the brush may be safely worked. Thus the product $V_r \times$ current density is a constant, and usually has a value of about 55. This, however, depends to some extent upon the pressure employed to keep the brush up to the commutator face; for the contact resistance varies both with this and with the current density. With ordinary grades of carbon brush the pressure employed should be about $1\frac{1}{4}$ lb. to $1\frac{1}{2}$ lb. per square inch; but with brushes made of almost pure graphite the pressure may be considerably increased, up to 3 or even 4 lbs. per square inch, because the material of the brush forms a natural lubricant. Fig. 82 shows the variation of properties due to changing pressure of the well-known Morganite 'Link' 1 brush which is so constructed as to have a high resistance across the brush face, but a low resistance lengthwise. Fig. 83 shows a curve at $1\frac{1}{2}$ lb. per square inch for the Battersea carbon brush, type C, which is a first-rate grade of hard carbon brush. With special brushes, such as Morganite, the value above given for the product of V_r and current density will not, of course, hold, but they should be judged from curves such as Fig. 82. The following table of well-known English and American brushes gives the makers' estimate of current

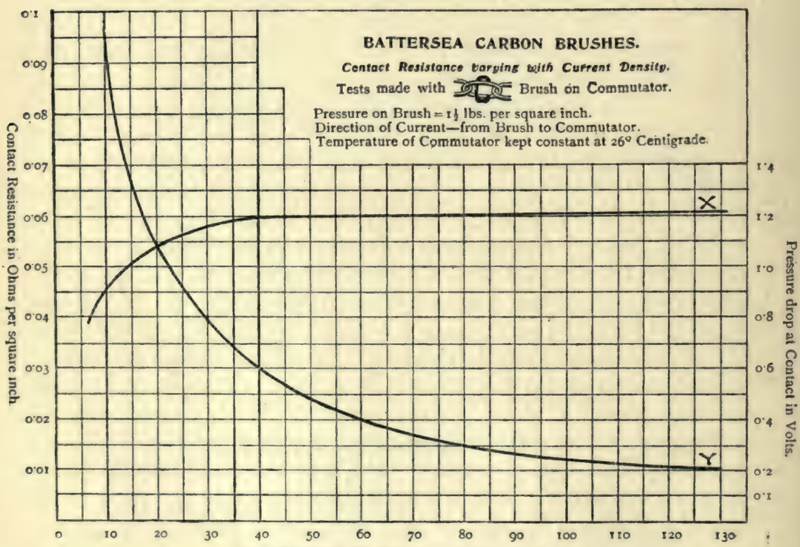
* *Proc. Amer. I.E.E.*, vol. 24, p. 420; see also Carter's paper in *Elec. World* (N.Y.), March 31, 1910.



Current Density in Brush in Amperes per square inch

Curve X = Pressure drop with a pressure of 3 lbs. per square inch on Brush.
,, Y = Contact Resistance with a pressure of 3 lbs. per square inch on Brush
,, a = ,, ,, ,, ,, 2 ,, ,, ,, ,,
,, b = ,, ,, ,, ,, 4 ,, ,, ,, ,,

FIG. 82.



Current Density in Brush in Amperes per square inch.

X = PRESSURE DROP CURVE. Y = CONTACT RESISTANCE CURVE.

FIG. 83.

density, together with the author's estimate of the value of V_r which they will deal with satisfactorily, and of the coefficient of friction, and of the pressure which should be used with them :—

TABLE VI.

Brush.	Current density, amps. per sq. inch.	V_r.	Coefficient of friction.	Pressure, lbs. per sq. inch.
Battersea A	40	1·4	0·22	1·5
" B	45	1·2	0·25	2
" C	30	1·8	0·28	1·5
Partridge	35	1·5	0·25	1·5
National Columbia ...	28	1·9	0·28	1·5
Le Valley Vitæ	25	2	0·3	1·5
National Graphitized ...	55	1	0·22	2

Pressure Drop due to Contact Resistance.—The passage of the current from brush to commutator inevitably produces a pressure loss, which, however, is curiously constant for wide variations in current density, as is shown in the curves X in Figs. 82, 83. For the ordinary range of carbon brushes, at their appropriate current densities, the voltage drop varies from 1 to 1·2 volt according as the brush is of low or high resistance; *i.e.* suitable for a low or a high value of V_r.

Machine Dimensions limited by Reactance Voltage.—The formulæ on p. 128, taken in conjunction with Table VI., show that, if resistance commutation be relied upon, the output is limited by the type of brush adopted. Thus, with circular poles, a lap-wound armature, and Battersea C type brushes

$$\text{K.W. output per pole} = \frac{\text{pole diameter} \times 1{\cdot}8}{0{\cdot}55 \times \text{t.p.s.}}$$

$$\textit{i.e.}\ \text{K.W. output} = \text{poles} \times \text{pole diameter} \times 3{\cdot}26$$

with one turn per commutator section.

Substituting pole diameter × poles = $1{\cdot}5\ D.\lambda = 1{\cdot}72\ D$

$$\text{we get } D = \frac{\text{K.W.}}{5{\cdot}6}$$

$$\text{or, for square poles, } D = \frac{\text{K.W.}}{4{\cdot}5}$$

with average magnetic densities.

These formulæ, it must be remembered, are derived from, and dependent upon, certain magnetic densities. By forcing up the

densities, the denominator in the above fractions may be increased. The average value for machines of about 200 K.W. would be, according to makers' catalogues, about 6·5; but for 100 K.W., 5·5 would seem to be about the usual figure.

It will be noticed that the speed does not enter the above equations, so that it is impossible to compare the formula directly with the limiting values worked out on pp. 22 and 84, as prescribed by other conditions. If we work out the concrete case on pp. 23 and 85 under these conditions, it will form a fair comparison.

Example.—The machine is to give 200 K.W. at 500 volts and at 400 r.p.m.

$$D = \frac{\text{K.W.}}{5{\cdot}6} \text{ for circular poles} = \frac{200}{5{\cdot}6} = 35{\cdot}7''$$

$$\text{say} = 35''$$

$$\text{Pole diameter} \times \text{poles} = 1{\cdot}72D = 60$$

Whence with four poles

$$\text{Pole diameter } (= \text{armature core length}) = 15''$$

$$D^2L = 18{,}500$$

With six poles,

$$\text{Pole diameter} = 10''$$

$$D^2L = 12{,}200$$

Thus a six-pole machine would be cheaper than a four-pole machine, if resistance commutation were relied upon; and in either case the machine is larger than considerations of temperature rise alone would dictate (cf. p. 86). *In this way is the choice of the number of poles influenced by commutation.*

Minimum Size for Large Generators.—It has been said that increasing the magnetic densities will result in a smaller machine for the same reactance voltage, and the above example shows that average densities lead to greater armature dimensions than are dictated by temperature rise alone. These conditions have tempted designers to force up the magnetic densities until the size of the machine, without inter-poles, corresponds almost with the limiting temperature. There is, of course, a limit to this, but it is not reached before $D = \frac{\text{K.W.}}{9{\cdot}5}$, which corresponds to a pole-face density of about 65,000 and a value of $V_r = 2$. The advantage in the use of inter-poles, then, obviously consists in reduced densities and field ampere-turns.

Commutation Losses.—On p. 86 the sources of heat in the commutator have been enumerated. Since carbon brushes are now universally adopted, the particulars just given of well-known makes can be used for estimation of the commutator losses. The electrical loss, for instance, can from Fig. 83 be directly determined, since it is the product of the current collected per brush arm, the

number of arms, and the pressure drop at the contact. Failing such a curve to work from, a good average value for the resistance per square inch of contact surface is 0·03 ohm.

The number of square inches of brush surface is dictated by the current density which can be employed for the particular brush selected; so that from these two factors the watts lost electrically can be computed as follows:—

Current per brush arm = total current $\div$ number of positive or negative arms

Area of brush surface per arm = current per arm $\div$ current density

Resistance of contact per arm = 0·03 $\div$ area per arm

Loss at each brush arm = (current per arm)^2 $\times$ resistance per arm

Total electrical loss = number of arms ($+$ and $-$) $\times$ loss per arm

The electrical loss, then, may be expressed as

$$\text{Total current} \times \text{current density at brush} \times 0{\cdot}06$$

but it is better to take it from curves like Fig. 83. From Fig. 83 it would be $2{\cdot}4 \times$ total current.

Friction Losses.—The coefficients of friction for the various types of brush are given in Table VI. If V_c be the peripheral velocity of the commutator surface in feet per minute, the friction loss is, in watts—

$$\text{Brush pressure per sq. in.} \times \text{total brush area} \times \text{coefft.} \times V_c \times \frac{746}{33000}$$

$$= 1{\cdot}5 \, \frac{2 \times \text{total current}}{\text{current density in brush}} \times 0{\cdot}0226 \, V_c \times \text{coefft. of friction}$$

if $1\frac{1}{2}$ lb. per sq. inch be allowed; and this is

$$= 0{\cdot}068 \, \frac{\text{total current}}{\text{current density in brush}} \times V_c \times \text{coefft. of friction}$$

or, assuming 30 amperes per sq. inch as the brush-current density,

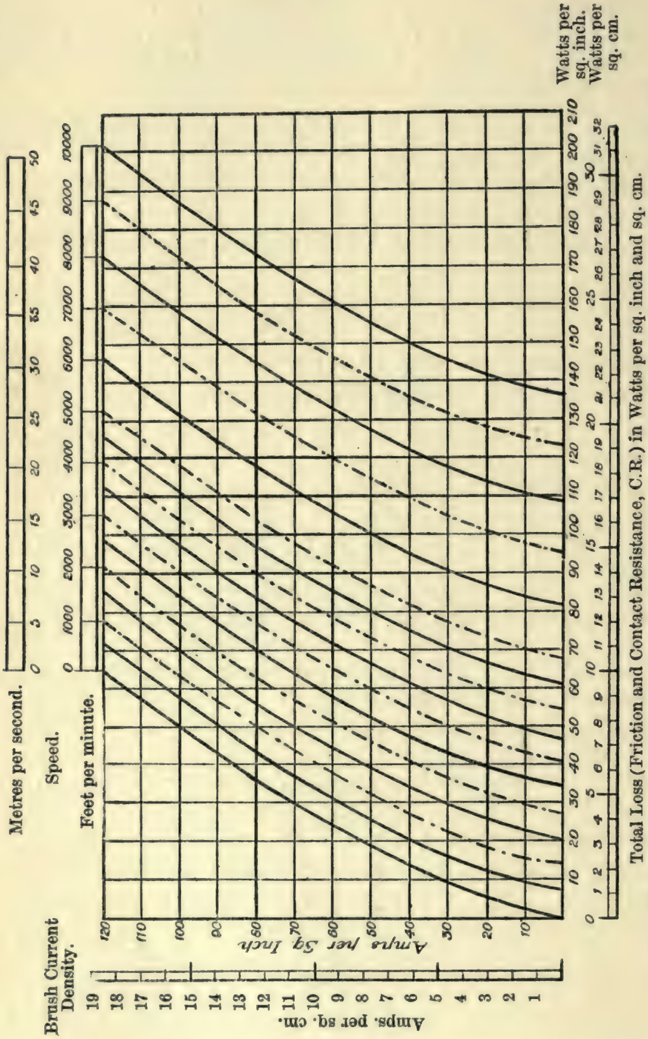
$$\text{Friction loss} = 0{\cdot}00226 \times \text{total current} \times V_c \times \text{coefft. of friction.}$$

When the brush constants are accurately known, curves like those of Fig. 84 can be constructed, from which the losses can be read off at once.

Commutator Dimensions.—Adding together the electrical and friction losses, the cylindrical area of the commutator face is immediately determined, as shown on p. 86, from temperature rise limits; whence the product diameter $\times$ length of face is obtained. The value of diameter and face length are then dependent upon the following considerations:—

1. The weight of the commutator should be as small as is

consistent with good commutation and a fair life. Now, the total depth of segment radially is nearly constant; it varies between $1\frac{1}{2}$



NOTE.—Both positive and negative values of Contact Resistance allowed for at a constant temperature of 45° C.
Pressure per sq. inch = 3lbs., or 200 grms. per sq. cm. Coefficient of friction = 0·2.

FIG. 84.—MORGANITE BRUSH CONTACT LOSSES IN WATTS PER SQ. INCH WITH VARYING DENSITIES AND SPEEDS.

and 2 inches, according to the commutator diameter. It follows that the commutator diameter should be as small as possible.

2. The diameter of the commutator should be less than the diameter of the armature measured at the bottom of the slots. It must,

however, be great enough to allow of a minimum thickness per segment of at least 0·15 inch + an insulation thickness of 0·03 inch.

3. The axial face-length is frequently determined by the brush collecting surface independently of temperature rise; for if the number of segments to be covered by one brush be fixed, the length of the commutator varies directly as the current to be collected.

Number of Segments covered by the Brush.—This is a question that has never been satisfactorily settled. The author usually limits it to the number of coils per slot. If the brush cover too many segments, bad contact ensues; or sometimes the coil is so long a time under the brush as to cause it to enter well under the pole-tip before it is thrown into circuit again, and in these circumstances good commutation is almost impossible. On the other hand, the brush should be at least as wide as one commutator section. In practice, the average number of segments covered is three, and the average brush thickness is about $\frac{5}{8}$ inch (cf. p. 169).

Number of Brushes per Arm.—There should not be less than two brushes per arm. An accident to one does not then throw the machine out of service. Too great an axial length per brush means poor contact. An axial length of 1 inch to $1\frac{1}{2}$ inch per brush is usual.

Various Commutation Limits.—Besides sparking due to too heavy a reactance voltage, various other troubles may appear in machines made without inter-poles. Of these the chief are: (1) Glowing of the brush tip due to excessive current density there. (2) Spitting, which consists of little explosive sparks appearing occasionally. (3) Picking up copper by the brush. The first seems to be avoided if the armature ampere-turns per pole do not exceed 6500. The second is apparently a faint effect of reactance-voltage combined with heavy armature-reaction, and can therefore be avoided. The third is avoided by keeping the brush to its proper density in amperes per square inch.

Example of Commutator Design.—200 K.W., 500 volt, 400 r.p.m., six-pole lap-wound generator; without inter-poles.

Diameter of armature = 35″.

Maximum convenient diameter of commutator = 30″.

Peripheral speed, assuming 28″ diameter, 2940 ft. per min.

Electrical loss (p. 133) = $2{\cdot}4 \times 400 = 960$ watts.

Coefficient of friction = 0·3.

Friction loss (p. 133) = $0{\cdot}00226 \times 400 \times 2940 \times 0{\cdot}3 = 800$ watts.

Total losses = 1760 volts.

Cylindrical surface = 440 sq. in.

Diameter × length = 140 sq. in.

Face length from temperature rise = 5″.

Number of segments possible $\frac{\pi \times 28}{0{\cdot}18} = 488$.

Width of brush = $\frac{3}{4}''$.
Amperes per arm = 133.
Brush area per arm (with current density = 30) = 4·4 sq. in.
Axial length of brushes, $4 \cdot 4 \times \frac{4}{3} = 5 \cdot 9''$.
Axial length of face, allowing for six brushes, each 1″ long with clearances = 8″.

Thus in this instance the commutator face length is determined by the current to be collected rather than by the surface necessary for radiating purposes.

CHAPTER X

INSULATION

Insulating Materials.—It is an unfortunate fact that those materials which are most effective as insulators usually lack some mechanical quality, such as toughness or flexibility. In consequence, to meet the conditions of dielectric and mechanical strength, the insulation adopted must usually be composite; *i.e.* made up of two or more materials, one chosen for mechanical strength, the other for dielectric strength.

I. Insulators with Good Mechanical Qualities.—Of materials possessing the former qualification, especially toughness, the chief are: the various forms of *fibrous or cellulose material,* such as paper, press-board, fullerboard, press-spahn, vulcanized fibre, cotton fabrics, and so on. None of these materials has a very high dielectric strength, because they all absorb moisture to some extent, and most of them are easily damaged by high temperature. Some idea of their relative dielectric values may be gleaned from the following table of disruptive strengths, *i.e.* of alternating electrical pressure with a sinusoidal wave-form at which puncture occurs:—

TABLE VII.

Dry material,* 0·04″ thick.	Disruptive strength (R.M.S. volts).
Brown paper . .	7,000
Red rope paper .	6,800
Express paper . .	7,000
Manila paper . .	6,000
Press-spahn . .	9,000
Horn fibre . .	12,000
Vulcanized fibre .	6,000
Dry wood (maple) .	600

* Dried in a vacuum oven for four hours at 75° C., and tested, when cold, within half an hour afterwards. Compare, however, *Electrician,* 1905, p. 949.

Besides the use of cotton for covering wires, which is referred to more fully a little later, various cotton and flax fabrics are made up and impregnated for use as insulating materials. These are broadly spoken of as varnished linen, varnished long cloth, varnished canvas, and so on. Many are sold under special trade-names, such as Empire cloth, etc. They are very effective as insulators, and are nearly all prepared by drying the woven fabric in a vacuum oven and steeping it in the varnish; afterwards the excess varnish is drained off, and the material dried in a stove to which an ample supply of air is supplied to oxidize the varnish thoroughly. Such preparations have a disruptive strength of from 7000 to 10,000 volts (R.M.S.) in the case of sheets 10 mils thick.

Too much stress must not be laid upon the value of the disruptive strength, for so much depends upon the exact test conditions that the figures are more valuable relatively than actually. In all the tests given above, moisture was driven out before test; for, since there is not one of the materials in the whole list which does not absorb moisture to some extent, all before use should be dried and impregnated with some insulating varnish. This treatment may, or may not, considerably increase the disruptive strength measured under average *atmospheric* conditions, but there is no questioning the enormous increase in safety under ordinary *working* conditions. The increase in the disruptive strength of red rope paper, for instance, seems to be at least 70 per cent., while that of press-spahn is little changed; yet for practical safety both require impregnating.

Again, the duration of the test has much to do with the disruptive strength. For if the test pressure be left on, it almost always sets up local heating, which causes the material to break down at a much lower value than is shown in the table. Thus an increase of temperature of 30° C. has been known to reduce the dielectric strength 50 per cent.

Varnishes.—For impregnating cellulose and other materials many excellent varnishes are obtainable. Most of these seem to be founded upon boiled linseed oil, and there is considerable divergence of opinion as to the desirability of this compound on account of the possible presence of acid. The varnish known as "Armalac" seems to be free from linseed oil, and is said to be a solution of paraffin wax of a specially high melting-point. If this be the case, it overcomes many of the disadvantages of other varnishes, and the author has certainly found it very satisfactory. Examples of the use of these varnishes are given later in connection with typical insulation arrangements. Shellac varnish is now almost entirely given up because of its inflexibility when dry.

II. Insulators with Very High Dielectric Strength.—Of materials possessing a high dielectric strength, but poor mechanical

qualities, the best known is "mica." Many varieties of this mineral are found, differing from one another in colour, hardness, and insulating properties. The disruptive strength for plates 0·04 inch thick appears to be from 20,000 to 200,000 R.M.S. volts, so that its capacity for resisting breakdown is very high. It is further practically fire-proof, and suffers only from its brittleness and liability to come away in thin laminæ or flakes. To overcome this, it is made up into various special forms by being cemented on to a backing, as mica-paper, mica-longcloth, mica-canvas. It is also made up with shellac or other cement into "micanite," and as such may be moulded while hot into almost any desired form.

These mica preparations are excellent insulators, having disruptive strength somewhat as follows:—

TABLE VIII.

Material made up to 0·04″ thick.	Disruptive strength (R.M.S. volts).
Micanite . .	30,000
Mica-canvas . .	2,500
Mica-longcloth .	2,500
Mica-paper . .	15,000–20,000

Porcelain is another fire-proof material possessing high dielectric strength, but because of its brittleness it is only suitable for such purposes as bushes, fuse handles, and the like. *Asbestos* is a fair insulator, and also fire-proof, but it is very friable; it is used, as will be seen later, for field-coils, but requires impregnation to render it satisfactory.

Many rubber compounds are used, such as *vulcanite* and *ebonite*. These, especially the latter, are excellent insulators; but they suffer from the disadvantage that if made flexible they soften at a very low temperature, and if made hard they are usually brittle.

Insulation of Round Wires.—Insulated wire is made by spinning over the wire a coating of cotton or silk. Cotton-covered wires are made in three grades: (1) single cotton covered (S.C.C.), (2) double cotton covered (D.C.C.), (3) fine double cotton covered. Triple-covered wire is also made for special purposes. Silk-covered wires are used in practice with a double covering only. The allowances for thickness of these coverings vary with the diameter of the wire itself, but may generally be taken as follows:—

1. *S.C.C.*—For all sizes to No. 20 S.W.G. add to the diameter of the wire 5 mils.

Nos. 19 to 13 S.W.G., add 8 mils.

Nos. 12 and larger, add 10 mils.

Double Cotton Covered (ordinary).—For all sizes to No. 18 S.W.G., add to the diameter of the wire 10 mils.

Nos. 17 to 13, add 12 mils.

Nos. 12 and larger, add 14 mils.

Fine Double Cotton Covered.—All sizes to No. 20 S.W.G., add 6 mils.

Nos. 19 and 18, add 8 mils.

Nos. 17 to 13, add 10 mils.

No. 12 and larger, add 12 mils.

A double silk covering adds from 4 to 8 mils to the diameter of the wire according to size.

It is evident that where the coil consists of a large number of fine wires (as in high-voltage machines), a much higher space-factor may be obtained by using the fine D.C.C., or, in special cases, silk covering; and the extra expense entailed is often more than compensated for by the larger output obtained.

Wire manufacturers are usually willing to vary the covering to quite a considerable extent to suit special requirements, but of course it is difficult then to carry a reasonable stock.

Heavy wires, rectangular conductors, and strip, are usually covered with a fine close braiding, which is in general from 5 to 8 mils thicker than ordinary double lapping. Sometimes a double-cotton covering is used as well, inside the braiding (cf. p. 206).

Choice of Coverings.—For field-coils with a voltage of not more than 100 per coil, S.C.C. is usually good enough, particularly if the coil be thoroughly impregnated after winding.

For armatures D.C.C. should always be used.

Space-factor.—The ratio of the nett sectional area of the copper in an armature slot to the actual area of the slot is called the *space-factor*. Similarly, in the case of coils, if a right section be taken through the coil, the ratio of the space occupied by the copper to that occupied by copper and insulation together is called the space-factor. Thus in Fig. 44 the space-factor of the coil is

$$\frac{\text{Number of turns} \times \text{sectional area of wire}}{l_c \times d_c}$$

The space-factor is thus a measure of the utilization of the winding-space, and cannot be greater than unity. The object of the designer should be to get the space-factor up without lowering the dielectric strength, and this can only be achieved by most careful and scientific use of insulating material. Naturally the space-factor will depend not only upon the method of winding, but also upon the size and sectional shape of the wire used. Strip will give a higher space-factor than round wire, and a low-voltage coil will have a higher

space-factor than a high-voltage coil. For preliminary calculations a knowledge of the probable space-factor is especially useful.

Space-factor of Field Coils.—This will obviously depend upon the type of coil used. Broadly speaking, two types only are made; in the one case the wire is wound upon a spool of metal, or of some special material, and in the other case the coil is wound and taped without any spool. These two types have already been illustrated in Figs. 41 and 43, p. 73. The second form is the cheaper to make, and usually occupies less room; but, as already shown, it does not so readily dissipate the heat generated in it.

For shunt field-coils the space-factor ranges from 0·2 in the case of a high-speed motor of 500 volts and 2 H.P. up to 0·6, or, in exceptional cases, 0·7 in the case of large generators for 200 volts.

When the space-factor is as low as 0·2 it usually pays to raise it by adopting wire with a special covering.

Series coils having fewer turns of larger cross-section naturally have a much higher space-factor, and a value as high as 0·75 may sometimes be obtained by winding copper strip edgewise.

Examples of Field-Coil Insulation.—Since it is impossible to lay down any hard-and-fast rule as to the arrangement of the insulating material about a field coil, the only course is to give for guidance reliable examples of modern practice. The whole subject of insulation is but in its infancy, and careful study of it will well repay any manufacturer of electrical machines. Below are cited instances of each of the two main types of coil previously referred to, and of the details of modern railway- and tramway-motor coils, which often receive rough treatment.

Example 1.—**Metal Spool** (Figs. 41 and 42).—The spool is lined with varnished press-spahn 10 mils thick, strengthened by one layer of empire cloth 5 mils thick in machines for 440 volts and over. Strips of tape $\frac{5}{8}''$ wide, zigzagged in and out of the last layers, are employed to fix the outer turns, and the inner end is brought up a special groove or hole made in the spool (cf. Fig. 131). The whole finished coil is dried in a vacuum oven, and then immediately treated with an insulating varnish, and afterwards baked. Sometimes a special brass terminal is bound into the coil, as in Fig. 131.

Example 2.—**Taped Coil** (Figs. 43, 85, and 124).—These coils are usually wound upon a wooden spool with loose ends, so that when completed they can easily be removed. The turns are kept in place by zigzag tapes throughout, and when removed from the spool the coil is usually lined with varnished press-spahn to protect it from mechanical abrasion, and coil and press-spahn are then bound all over with tape about $\frac{5}{8}''$ wide. The turns of the tape are allowed to overlap for about half their width; and two servings are put on, one

as a right-hand spiral, the other as left-hand spiral, so that they mutually interlock. The completed coil is dried in a vacuum oven, treated with insulating varnish, and baked, the three processes being carried out twice. The inner end is brought out carefully and specially protected by an insulating sleeve.

Example 3.—Where much vibration is likely and a high voltage is used, as in the case of traction-motors, more careful construction is adopted. Usually these machines are series wound, so that the conductor is a large wire, a cable, or a flat strip; and often it is necessary to render the coil fire-proof. The following instructions for such coils exemplify methods commonly adopted.

Railway - Motor Field - Coils. Fire-proof Construction. Strip Wound.—The turns are insulated by winding between them two layers of asbestos each 10 mils thick. The coil is then dried thoroughly, well varnished, and pressed whilst hot. Moulded mica corner and side pieces 40 mils thick are placed on the coil to protect it and to keep the turns from sliding over one another; and further protection is afforded by layers of mica and asbestos made up into washers about $\frac{1}{8}$ inch thick and placed over the pole tip and also under the yoke. The ends of the coil are usually thoroughly insulated; especially is it necessary to take care with the inner end, which on being led out crosses the other coil-layers, and must be insulated therefrom by strips of mica. Heavy cables have to be joined to the terminal ends of the coils, and the joint between these and the strip must be most carefully insulated.

Example 4.—**Traction Motor Field Coils—Ordinary Construction.**—The coil is wound from wire of the required size, and each layer is painted with a good enamel or varnish. The corners and the places where wires cross over are protected by asbestos $\frac{1}{16}$ inch thick. Where special protection is necessary, as at the first and last turns of the first layer of wire, a turn of string or rope may be wound parallel with the wire, so that any pressure comes on this and not on the wire itself. The same protection may be adopted in the last layer where the coil comes against the yoke. The coil should then be baked in an oven for twelve hours, or if a vacuum oven is available half this time will be sufficient. While still hot it should be plunged into insulating varnish and afterwards dried in a stove for ten hours, or in a vacuum oven for five hours, at a temperature of about 100° C. Inside the coil, and surrounding the pole, should be placed a liner of fullerboard $\frac{1}{32}$ inch thick, which has been previously treated with varnish and well dried to render it non-absorbent, and at either end there should be a washer of fullerboard or press-spahn $\frac{1}{16}$ inch thick. The coil may finally be taped all over and again treated with varnish, the usual care being exercised in the manner of bringing out the terminals.

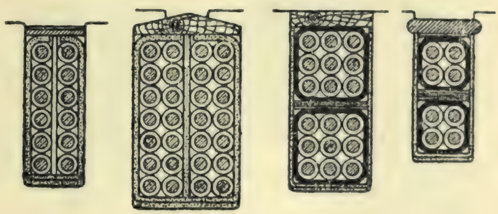
Space-factor of Armature Slots.—In this case also the value of the space-factor will depend upon a number of conditions, the chief of which are:—

(*a*) Type of winding and method of insulation.
(*b*) Shape of wire section, whether rectangular or round.
(*c*) Number of armature coils corresponding to one slot.
(*d*) Voltage and speed.

(*a*) *Type of Winding.*—We have assumed the winding to be always of the drum-type, with cylinder end connections and former-wound coils. The coils corresponding to one slot, separated by insulating strips, are usually bound together by wrapping with tape 10 mils thick, as mentioned and illustrated on p. 114. These units are then always dried in an oven (as described for field coils), impregnated with a good varnish, and dried again. Often this is repeated, and in some cases it is carried out three times. These units are then placed in the slots. Sometimes an insulating composite liner is placed in the slot, and sometimes this protection is put inside the taping and so forms part of the coil. In any case the insulation thickness between wire and core is rarely less than

35 mils	for	100 volt	machines	(1500 R.M.S. volts flash-test)
40	„	200	„	(2000 volts flash-test)
60	„	500	„	(3000 „ „)

Fig. 86 shows some examples of slot coil-windings, in which the distinction between the slot-lining and the taping, etc., round the



1 2 3 4

FIG. 86.—ARRANGEMENT OF WIRE AND FORMER WINDINGS IN SLOT.

coils is brought out. Table IX. gives some typical examples of composite slot-linings for various voltages.

TABLE IX.

SLOT LININGS FOR TAPED FORMER-WOUND COILS.

Volts of machine.	Press-spahn or fuller-board. mils.	Oiled cloth or paper. mils.	Mica or mica compound.	Press-spahn or fuller-board.	Insulation thickness.
Up to 150	10	5	—	10	2 × 25
„ 250	10	10	—	10	2 × 30
„ 600	12	12	14	12	2 × 50

Between the groups of coils themselves, or between the upper layer and lower layer of the end connections, varnished press-spahn 20–30 mils thick may be used.

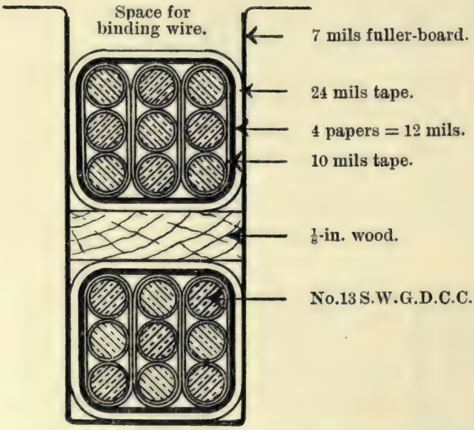


FIG. 87.—SECTION THROUGH SLOT OF A TRACTION MOTOR.

Not infrequently in modern motors the slot-lining is partly or wholly omitted. In such cases, of course, the coils themselves must be more heavily insulated, and Fig. 87 illustrates such an arrangement suitable for a 500-volt traction motor for tramway work. An armature so wound, if properly baked and impregnated, would withstand a test of 3000 volts for three minutes.

Another example of slot insulation in which no slot liner at all is used is as follows :—

The individual coils, instead of being separately taped, as in Fig. 87, are separated by shellaced press-spahn 10 mils thick.

The coils corresponding to one slot are bound together by taping,

as in Fig. 87, up to a thickness of 20 mils, and the two half coils in the same slot are separated by a strip of press-spahn 10 mils thick. This arrangement is for 220 volts.

(*b*) *Shape of Wire.*—The effect of the shape of the wire upon the space-factor of the slot is well illustrated in Fig. 88, giving the comparison between the slot dimensions for coils of the same copper area and of the same insulation, for round and rectangular conductors respectively. The saving obtained by using rectangular wire amounts to a difference in space-factor of from 15 to 25 per cent., 20 per cent. being a fair average. A very convenient winding may be made from a thin flat strip wound with the flat side parallel to the bottom of the slot. This arrangement is adopted in Adamson's crane-motors. Generally speaking, rectangular wires are more difficult to wind than round wires.

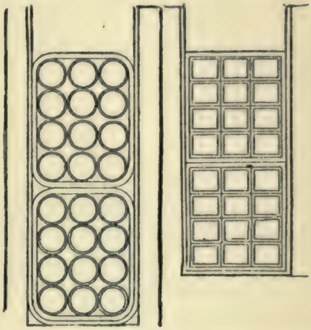


FIG. 88.—SPACE-FACTORS OF ROUND AND RECTANGULAR CONDUCTORS.

Bar Windings.—When the copper conductor is too large in section to admit of coil-winding, each turn is individually made on a former, and the armature is said to be

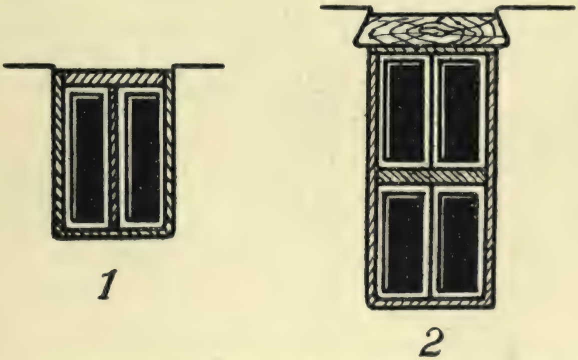


FIG. 89.—ARRANGEMENT OF BAR WINDINGS IN SLOT.

bar-wound. In these cases there are usually as many commutator-sections as armature-turns. Sections through slots so filled are shown in Fig. 89, and Table X. gives the usual slot-lining for various voltages.

TABLE X.

Volts of machine.	Press-spahn or fuller-board. mils.	Oiled cloth or paper. mils.	Mica or micanite.	Press-spahn or fuller-board.	Total insulation.
Up to 150	10	5	—	10	2 × 25
„ 200	10	12	—	12	2 × 34
„ 600	10	15	18	10	2 × 53

(*c*) *and* (*d*) *Number of Armature-Coils per Slot, Voltage and Speed.*—Each armature for a given voltage and speed has a number of coils which is fairly definitely fixed. The number of slots to be used, however, for each armature, admits of some choice; and in consequence the number of coils per slot is a matter for attention. Apart from the limitations referred to on p. 114, the number of coils per slot should be as large as is consistent with reasonable slot-dimensions; for the fewer the slots the better, as a rule, is the utilization of space.

On the other hand, the larger the number of turns per coil the worse is the space-factor, because the greater is the number of conductors per slot, and each conductor must be insulated. Thus the difference between four and two coils per slot corresponds to an increase in space-factor of about 12 per cent., *i.e.* a change from 0·4 to 0·45 about. On the other hand, doubling the number of turns per coil decreases the space-factor usually from 5 to 10 per cent., according to the size of wire involved.

General Values of Slot Space-factor.—Paying due regard to the above points, the following table gives average values of the space-factor for various outputs and voltages, with their corresponding testing pressure in R.M.S. volts (5 minutes' application):—

TABLE XI.

MACHINE VOLTAGE UP TO 150; TESTING VOLTAGE 2000.

K.W. output	5	10	15	20	30	40	50	60	80	100	200
Space-factor	0·28	0·34	0·38	0·4	0·43	0·45	0·47	0·48	0·5	0·51	0·52

TABLE XII.

MACHINE VOLTAGE UP TO 250 VOLTS; TESTING VOLTAGE UP TO 2500 VOLTS.

K.W. output	5	10	15	20	30	40	50	60	80	100	200
Space-factor	0·24	0·28	0·33	0·34	0·37	0·4	0·41	0·42	0·43	0·43	0·44

TABLE XIII.

MACHINE VOLTAGE UP TO 600 VOLTS; TESTING VOLTAGE UP TO 3500 VOLTS.

K.W. output	5	10	15	20	30	40	50	60	80	100	200
Space-factor	0·2	0·24	0·26	0·29	0·32	0·34	0·34	0·35	0·36	0·37	0·38

Insulation between Armature Laminæ.—This may be carried out (*a*) by pasting thin paper on to one side of each disc; (*b*) by Japanning or varnishing the discs; (*c*) by treating the discs with special preparations, such as "Insuline." The armature discs are usually stamped from metal about 18 to 20 mils thick, and the space occupied by the insulation is for method (*a*) about 12 per cent., for methods (*b*) and (*c*) about 8 to 10 per cent. The use of paper undoubtedly results in the least eddy currents; but the other methods are good enough, especially if a sheet of paper be used about every twenty stampings.

Insulation of Brush-Gear parts, Terminal Blocks, etc.—For such purposes as these, mechanical strength and small surface-leakage are of great importance. Consequently moulded blocks of various special compounds, such as ambroin, isolite, micanite, etc., have replaced the older materials like ebonite, vulcanized fibre and wood. Very many beautifully moulded and very convenient special insulators are on the market for these purposes. Porcelain is used in some special cases. For particulars of dimensions, etc., the reader is referred to the various makers' catalogues.

Commutator Insulation.—This may be divided into three parts, viz.—

1. That which insulates the sections from the clamping rings.
2. That which lies between the sections and the shaft.
3. That which insulates a section from its neighbours.

For the first and the third purposes, except in the special case of very low voltage machines, mica or micanite is always used. The end-rings for the former purpose are specially moulded to the correct form by micanite makers; but for the latter work it is usual to buy mica plates and split them to suit the thickness of insulation required, fixing them together by means of as little shellac varnish as possible.

For protecting the inside of the sections and separating them from the shaft, sometimes micanite, more often press-spahn, is adopted: for the only object of this liner is to protect the sections from short-circuit by the possible accumulation of dust, or by any little bits of metal left accidentally when putting the machine together. Figs. 111 and 112 give an idea of the usual construction; and the thicknesses of insulation employed are given below with their corresponding voltages.

THICKNESS OF MICA BETWEEN SEGMENTS.

Machine voltage.	Thickness in mils.
Up to 250	25
„ 600	25 to 35

The mica selected should be such as will wear at the same rate as the copper segments. The best Indian mica is probably as good as any for this purpose, and the author prefers it when of a greenish shade with or without greenish spots.

The micanite end-rings vary from 40 to 100 mils in thickness, according to the size of the machine; an average value is $\frac{1}{16}$ in. (= 62·5 mils).

Other details of insulation will be best studied from examples given in Chap. XI., and under the various machines worked out.

CHAPTER XI

GENERAL MECHANICAL CONSTRUCTION

In the preceding chapters, the considerations which govern and limit the proportions of the individual parts of continuous-current machines have been dealt with in some detail. It is now necessary to emphasize a few important points in the relative arrangement and construction of these parts; and since details of mechanical construction are largely in the discretion of the designer, where no theoretical laws as to strength apply, dimensioned examples will be given to form a safe guide.

I. Field-Magnets.—It has already been shown (pp. 5–7) that the general mechanical construction of the field-magnet influences to no small extent the subsequent design calculations. For this reason, questions relating to the shape and fixing of the poles, to the form of the shoes and to the yoke material and shape have already been dealt with. Other details of field design will be found in subsequent examples, as, for instance, on pp. 208, 218. Where poles and shoes are made up of stampings, it is usual to bind the latter together by means of rivets, as shown in the small pole-stamping, Fig. 90, and in the completed pole, Fig. 91. These groups are then held in position by set-screws passing through the yoke and into the tapped stampings, as in Fig. 91; or into a rectangular or round wrought-iron nut pressed into a hole left in the stampings, as indicated in Fig. 90.

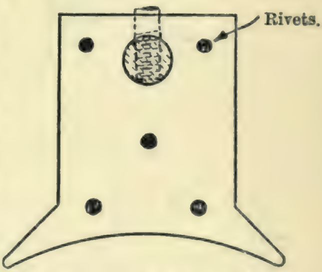


Fig. 90.—Pole-piece Lamination.

The two outermost of each group of stampings are usually of thicker material, so as to form supports for the entire group, as well as providing thickness for the countersunk rivet-heads (Fig. 91).

An illustration of the construction of a field magnet with solid

steel poles and laminated shoes is shown in Fig. 92, which gives details of a machine designed by the author.

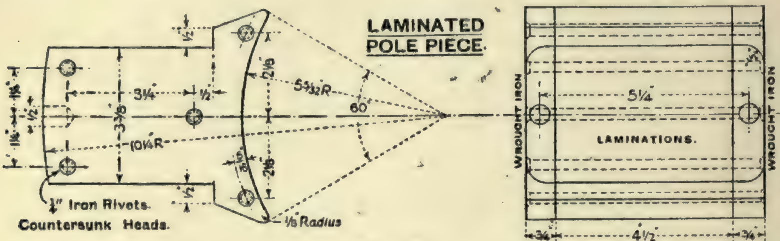


FIG. 91.

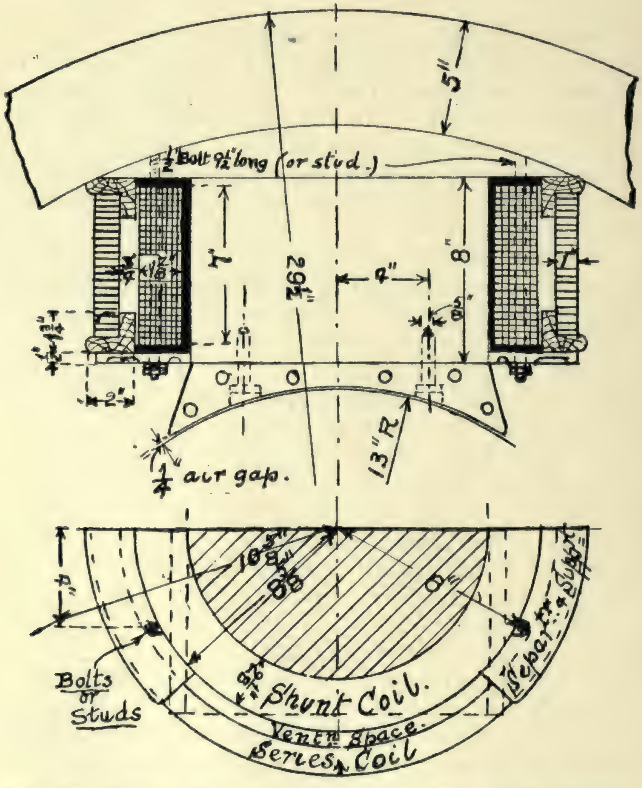


FIG. 93.—DETAIL SKETCH OF VENTILATED FIELD-COIL (CRAMP).

Fixing of Field Coils.—This depends entirely upon the construction and insulation of the coils, as illustrated, for instance, in Figs. 41,

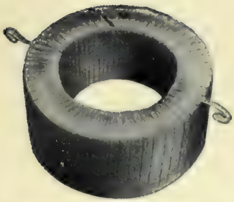


FIG. 85.—TAPED FIELD-COIL.

FIG. 97.—MACHINE WITH END-PLATE AND YOKE CAST TOGETHER (GENERAL ELECTRIC COMPANY).

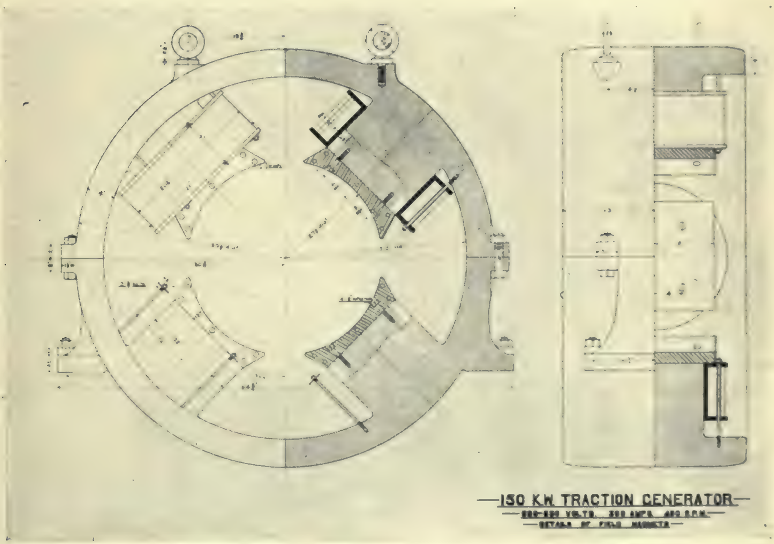


FIG. 92.—GENERAL ARRANGEMENT OF FIELD-MAGNET (CRAMP).

[To face p. 150.

42, 43, 45, and 85. The support for the coil is usually provided by the shoe, projections or brackets being fixed thereto, upon which, when protected by insulation, the coil rests. To prevent movement when in place, insulating wedges, often of wood, are used between the coil and pole; or a method of fixing like that illustrated in Fig. 92 may be adopted. A detail of this coil is given in Fig. 93, and it will be noticed that the series-coil is spaced apart from the shunt-winding, the supporting bolts passing through wooden sector separators.

Fixing of Interpoles.—Fig. 94 shows the ordinary method * of fixing interpoles as arranged by Lawrence Scott & Co. In order to find room for the interpole coil, sometimes either the main windings or the interpole-winding has to be specially shaped. In any case it is advantageous, when the main poles are circular, to displace the

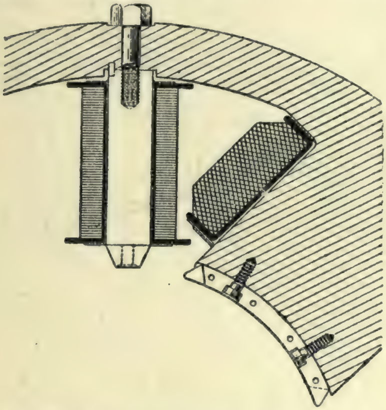


FIG. 94. INTERPOLE AND MAIN POLE (LAWRENCE SCOTT).

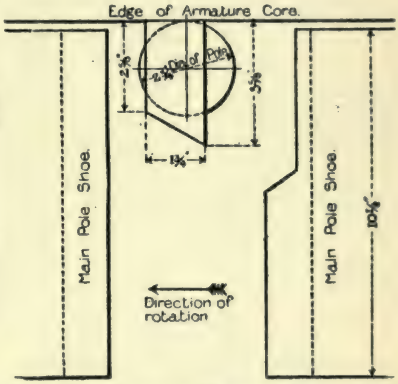


FIG. 95.—PHŒNIX PATENT INTERPOLE.

interpole with respect to the main pole, as shown in the plan, Fig. 95. It is also seen from this drawing that the interpole-shoe need not be of the same axial length as the armature. It is, of course, an advantage to have it shorter, as the leakage is thereby reduced, and this idea formed one of the main points in the original Pohl patent, which is now maintained by practically all the leading firms. To reduce the leakage still further, the Phœnix Dynamo Co. have patented the arrangement of cutting away that pole-shoe to which most leakage would take place, as shown in Fig. 95.

Machining of Field-magnets.—Field-magnets should be so designed as to reduce the machining to the lowest possible limit. To compass this end it is desirable to arrange so that as many surfaces as possible may be machined at one setting, due regard being paid to the tools available. Thus, where the poles are separate from the

* Cf. *Journal Inst. E.E.*, vol. 39, no. 186, p. 593.

yoke, the seatings for these poles may be bored or they may be planed; and though the former is cheaper, it often necessitates a large boring machine. Where the pole-seats are bored and the poles are made up of stampings, some makers do not machine the actual armature bore, as stampings can be bought sufficiently accurately finished to render this unnecessary. A double gain results, in the absence of machining, and the reduction of eddy currents. At the same setting, and often with the same tool, the pole-seatings (or the pole-faces), and the seatings for the bearings, may be machined. This necessitates cylindrical seatings for the bearing-standards; and if the latter be turned up with the bearing-bush as centre, absolute alignment of bearings, shaft, and armature bore is easily obtained. Of course this construction can only be applied in machines of moderate size. In Fig. 96, which refers to medium-speed direct-coupled generators as recommended by the American Standards Committee, flat standard seatings are adopted.

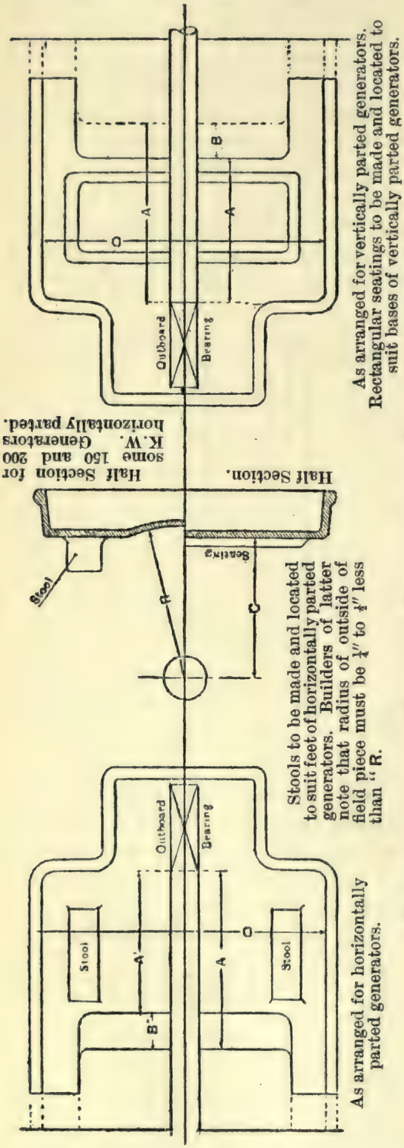


FIG. 96.—AMERICAN STANDARD BED-PLATES FOR DIRECT CONNECTION.

A careful examination of machines of well-known makers will show better than any description the various methods of simplifying the machining, which must be borne in mind when getting out a new design.

End-plates and Bearings.—In small machines, the bearings (usually of the ring-lubricated type, though sometimes

TABLE XIV.

TABLE OF SIZES, SPEEDS, AND STANDARDIZED DIMENSIONS OF DIRECT-CONNECTED GENERATING SETS.

(See Fig. 96.)

Capacity of unit, kilowatts.	Revolutions per minute.	Armature bore.		Diameter of engine shaft at armature fit.		Space occupied on shaft between the limit lines.		B, length of extension pieces, inches.	C, height of axis of shaft above top of base, inches.	E, inches.	D, width of top of sub-base, inches.	Key (A feather).				Holding-down bolts.	
		Centre crank engines, inches.	Side crank engines, inches.	Centre crank engines, inches.	Side crank engines, inches.	Long class A, inches.	Short class A, inches.					Width, inches.	Thickness, inches.	Depth in shaft at edge, inches.	Projection above shaft at edge, inches.	Diameter, inches.	Number.
25	310	4	4½	4 + 1/1000	4½ + 1/1000	30	25	5	23½	Flat.	48	1	¾	⅜	⅜	1	4
35	300	4	5½	4 + 1/1000	5½ + 1/1000	33	28	5	25	Flat.	54	1	¾	⅜	⅜	1	4
50	290	4½	6½	4½ + 1/1000	6½ + 2/1000	37	31	6	28	Flat.	60	1¼	¾	⅜	⅜	1	4
75	275	5½	7½	5½ + 1/1000	7½ + 2/1000	43	37	6	31	Flat.	66	1½	1	½	½	1¼	4
100	260	6	8½	6 + 1/1000	8½ + 2/1000	48	42	6	34	Flat.	72	1½	1	½	½	1¼	4
150	225	7	10	7 + 2/1000	10 + 2/1000	51	45	6	37¾	41¼	84	1¾	1¼	⅝	⅝	1¼	4
200	200	8	11	8 + 2/1000	11 + 2/1000	54	48	6	42¼	47½	96	2	1¼	⅝	⅝	1½	4

NOTE 1.—Five per cent. variation of speed permissible above and below speeds in table.

NOTE 2.—Distance from centre of shaft to top of base of outboard bearing may be less than C (to suit engine builder), though not less than possible outside radius of armature.

ball-bearings (Fig. 124) or wick lubricators are adopted) form part of the end plate, and are carried by the yoke. This construction gives excellent alignment, is cheap, and gives protection to the whole machine. One end-plate may be cast with the yoke, as in Fig. 97; this arrangement has the advantage of cheapness, but also the disadvantage that it is only possible to withdraw the armature at one end, unless the frame be split. The bearing in relation to its loose end-plate may be arranged as in Fig. 98 or as Fig. 99. In the latter, E is merely a screw to which the cap-chains may be attached.

In larger machines the bearings, also of the ring-lubricated type, are more usually carried by the bedplate, as is sufficiently illustrated

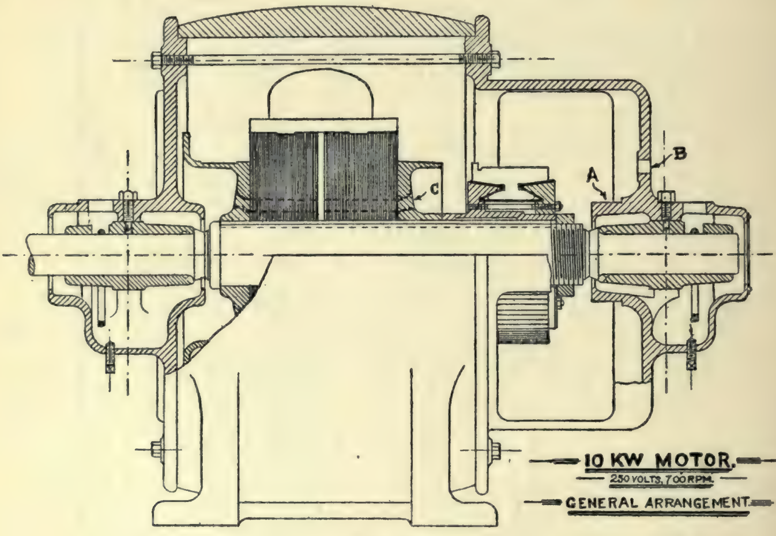


Fig. 98.

in Figs. 96 and 100. The number of lubricating rings is generally two, but bearings less than 6 inches long sometimes have but one. The bearing-bush itself is sometimes rigidly fixed in its shell, but more often it is arranged so that it can swivel slightly, as in Figs. 98 and 99. In small machines it is constructed of hard brass, or of gun-metal, or even of phosphor-bronze. In large machines it is constructed of cast iron and lined with white metal. Typical cases of the former are seen in Figs. 98 and 99, while Fig. 101 shows an arrangement of the latter class.

Proportions of Journals and of Bearings.—From the examples already given, and from the shaft calculations given later, the proportions of dynamo journals can be ascertained. The strengths

allowed, as well as the bearing surfaces, are usually far in excess of those prescribed by pure theory, and are more generous than mechanical engineers use under similar circumstances for other machinery. Reasons for this are to be found in the importance of keeping the armature central; in the constant running required; in the relatively high speed and large weight of the armature; in the liability to sudden strain if a

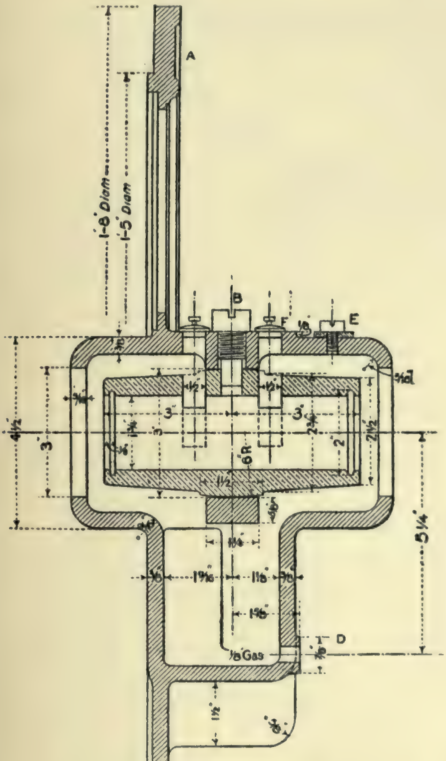


FIG. 99.—DETAIL OF MOTOR-BEARING (JONES).

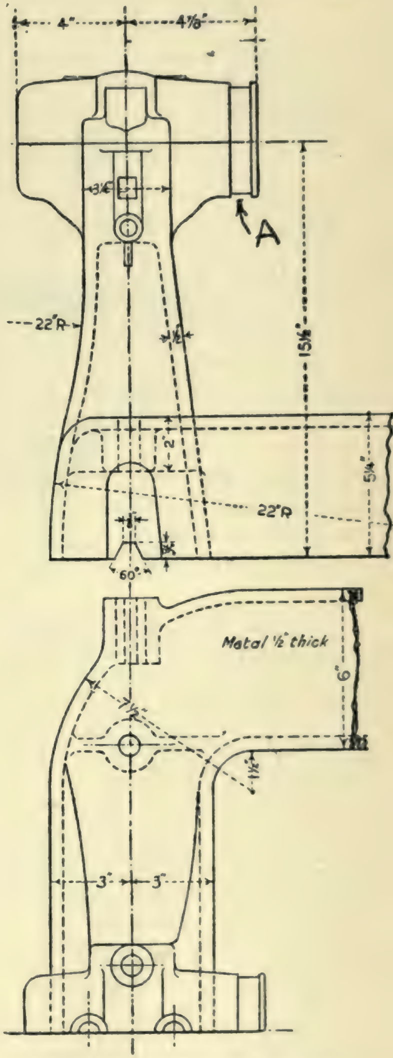


FIG. 100.—DETAIL OF DYNAMO-BEARING (JONES).

short circuit occur; in the small attention given and the absolute necessity for freedom from breakdown. The ratio, journal-length

to journal-diameter varies widely; but on an average may be taken at 3 to $3\frac{1}{2}$.

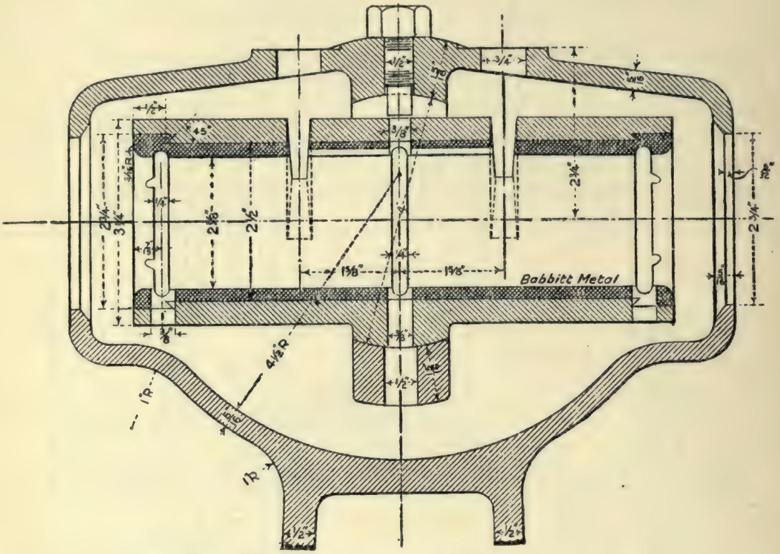


FIG. 101.—SECTION THROUGH PART OF FIG. 100.

Shafts.—Dynamo shafts are always made largest in the middle, stepped down to accommodate the commutator bush, and again at the journals (see Fig. 98). Oil-throwers are provided at the inner end of each journal, and shoulders to limit the end movement. The diameter of the shaft at the journals is sometimes calculated from a formula of the type—

$$\text{Diameter of shaft} = \text{constant}\sqrt[3]{\text{H.P.} \div \text{revs. per minute}}\,*$$

in which the constant depends upon the material used, and for steel is usually taken as 7. Such calculations are not very satisfactory, as they are based upon torsional forces only, the constant being taken large enough to cover any possible bending. The pure twisting moment on the shaft may be calculated from the usual expression—

$$\text{Twisting moment in inch-pounds} = 63{,}000\ \text{H.P.} \div \text{revs. per minute}$$

Bending Moment.—Some designers consider only the bending moment due to armature and commutator. They assume that the weight is concentrated at the centre, and that the shaft acts as a beam supported at the bearing centre at either end. The shaft is then designed so that the deflection due to this load is less than

* Cf. "Unwin's Machine Design," 13th Ed., vol. i. p. 214.

3 per cent. of the air-gap length. Thus, according to the usual formula for beams—

$$\text{Deflection at centre in inches} = \frac{4 \,.\, \text{weight} \,.\, (\text{length})^3}{3\pi a \,.\, (\text{diameter})^4}$$

The weight in this formula is that of armature and commutator in tons; the length is the distance from bearing centre to bearing centre in inches; and the diameter is that of the shaft centre in inches. a is the modulus of elasticity, usually taken at about 12,000 for steel shafts.

This method gives reliable results, although it does not take into account all the factors, or even all the bending forces. For instance, if there is a deflection the armature will be out of centre, and a magnetic pull will result. This can be approximately calculated, but the extra deflection due to it is usually far less than the reduction of deflection due to the stiffness given by the armature-spider and commutator-sleeve; also, since the load has been considered as concentrated when it is really distributed, a further factor of safety is added. In any case, to reduce the shaft to the smallest theoretical diameter would be a very short-sighted policy.

Combined Bending and Twisting.—If by an approximation like that in the previous paragraph the bending moment can be calculated, it is usually found that the twisting moment is negligible beside it. When this is not the case, the equivalent twisting moment can be found from the usual expression *—

$$T_1 = B + \sqrt{T^2 + B^2}$$

in which T_1 = resultant or equivalent twisting moment in inch-lbs.,
B = bending moment,
T = pure twisting moment as calculated on p. 156.

Whence, a formula similar to that on p. 156 may be obtained, giving: diameter of shaft = constant $\sqrt[3]{T_1}$, and the constant corresponding to the value 7 in the previous equation is 0·175. The value of B in inch-lbs. is often taken as weight of armature and commutator multiplied by $\frac{1}{4}$ of the length between bearing centres.†

It should be remembered that there is a critical speed for all shafts at which "whipping" is set up. This, however, rarely occurs at less than 2000 revs. per minute, and consequently is of importance for very high-speed machines only. It is not within the scope of the present work.

In the absence of practical experience, the reader is advised to calculate his shaft as carefully as possible from the formulæ given, and then to compare his results with such examples as those in Table XV.

* "Unwin's Machine Design," 13th Ed., vol. i. p. 215.

† Goodman, "Mechanics applied to Engineering," p. 364 and Chap. XIV.

TABLE XV.

Kilowatts.	Revolutions per minute.	Journal diameter in inches (pulley end).
4	1000	$1\frac{1}{2}''$
5	1000	$1\frac{3}{4}''$
5	600	$1\frac{7}{8}''$
10	1000	$1\frac{7}{8}''$
14	850	$2\frac{1}{8}''$
20	600	$3''$
60	500	$3\frac{1}{4}''$
100	200	$5''$
250	125	$6''$

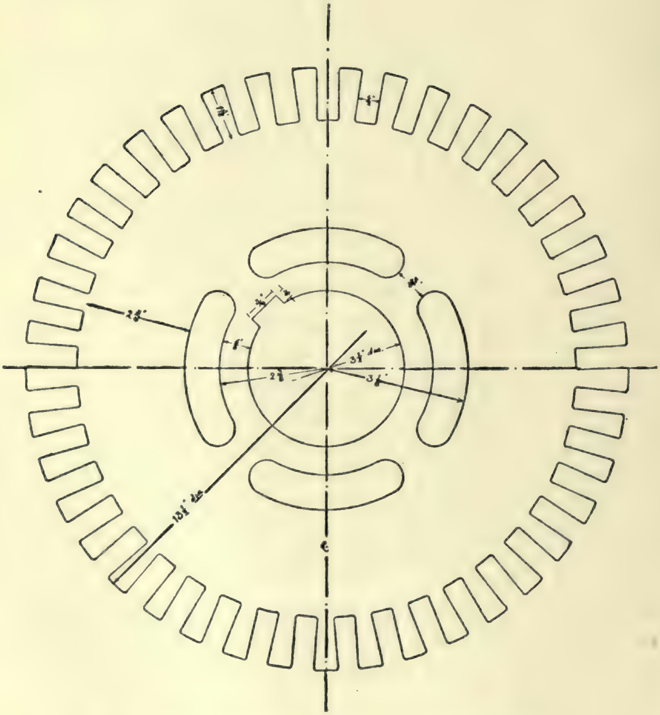


FIG. 102.—TRACTION MOTOR ARMATURE DISC.

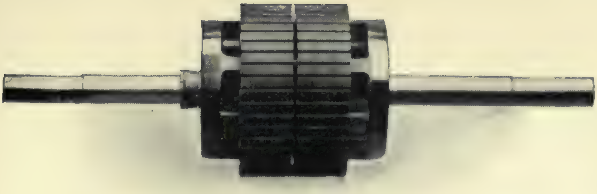


FIG. 104.—ARMATURE-CORE, END-PLATES AND VENTILATING DUCTS (VERITY'S, LTD.).

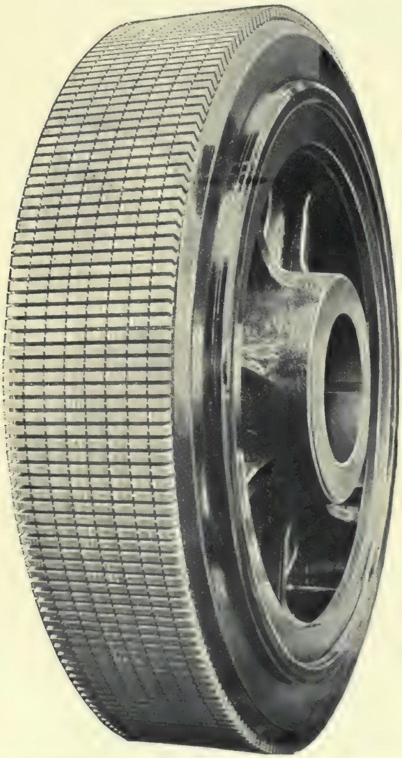


FIG. 107.—ARMATURE-CORE AND SPIDER (BRITISH WESTINGHOUSE CO.).

[To face p. 158.

The various steps and collars on the shaft of a small machine are seen in Fig. 98.

Armature Construction.—In very small machines the stampings of which the armature is composed are threaded straight on the shaft and clamped in position by suitable end plates (Fig. 98). Wherever possible, however, the stampings are threaded upon a 4-, 6-, or 8-armed "spider," whereby ample ventilation to core and windings may be secured. In machines with armatures of about 12 inches diameter there is hardly room for a spider, but there is room for more iron than is necessary. In these cases ventilation holes are made in the stampings, as illustrated in Fig. 102, for a

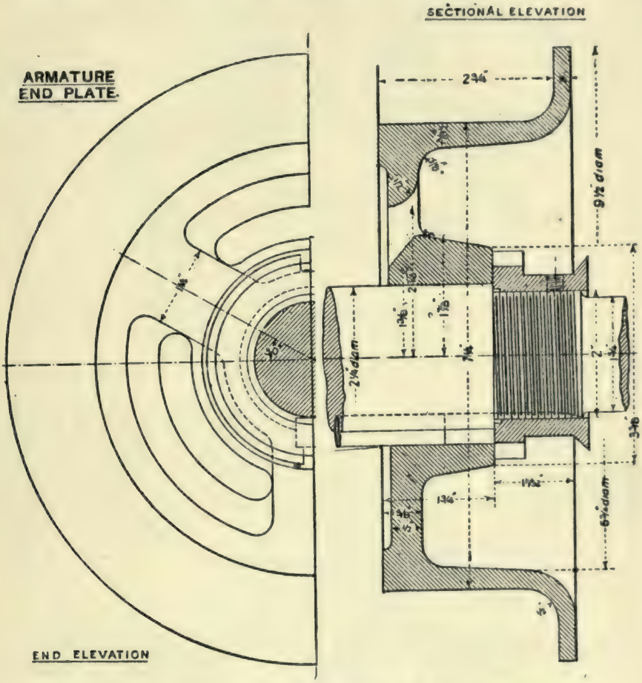
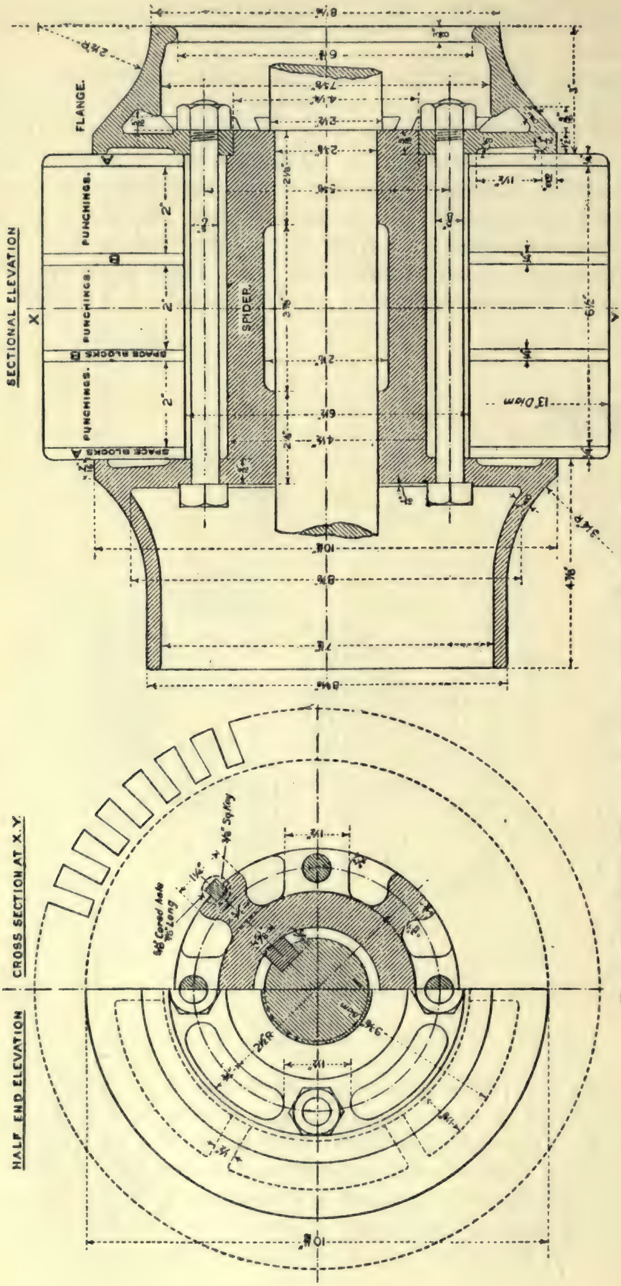


FIG. 103.—END-PLATE OR "CORE-HEAD" (JONES).

traction motor disc; examples of end-plates and the thick end supporting stampings of such armatures are shown in Fig. 129, p. 215, and again in Figs. 103 and 104. An armature of medium size mounted upon a spider is shown in Figs. 105 and 106, and a method of arranging still larger armatures is given in Fig. 107. In all cases the ventilating spaces are clearly shown. In Fig. 98 the passages through the core are dotted and marked with the letter C.



FIGS. 105, 106.—DETAILS OF ARMATURE (BRITISH THOMSON-HOUSTON CO.).

To drive the discs keys or feathers are used. In Fig. 98 the long feather is clearly shown; and in Fig. 105, there are two keys, one on the end of a spider arm to drive the discs, and another in the shaft to drive the spider.

Radial Ventilation through Armature.—The stampings of which the armature is composed are separated about every three inches to allow of the passage of air, as is shown in the illustrations. These passages or "ducts" are arranged by means of radial projections affixed to a stamping, or by a light ribbed cast plate riveted to the stamping next to the duct (Fig. 108). The projections in the latter

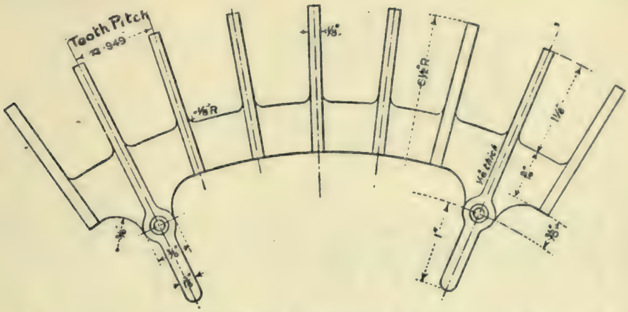


FIG. 108.—DISTANCE-PIECE (BRITISH THOMSON-HOUSTON CO.).

case reach right to the end of the teeth, and the ducts are from $\frac{1}{4}$ in. to $\frac{3}{8}$ in. wide.

Strength of Spider Spokes.—For a steady load it is easy to calculate the section of the spider arm. For the arm or spoke may be considered as a beam loaded at its outer end with a weight equal to its share of the pull at the armature periphery. The total value of the latter is evidently

$$\frac{\text{HP. } 6600}{\pi \mathrm{D} n}$$

Let the radial distance from the smallest section of the arm to the armature surface be l_1, and let the section of the arm be rectangular. Then, if the number of spokes be n_s—

$$\frac{\text{H.P.} \times 6600}{\pi \mathrm{D} n \,.\, n_s} l_1 = \text{safe stress} \times \frac{b_1 \times h_1^2}{6}$$

where $b_1 =$ smallest width of spoke parallel to shaft
$=$ length of armature, practically $= \mathrm{L}$ inches,
$h_1 =$ thickness of spoke at its smallest section at right angles to the shaft in inches.
and $n =$ revolutions per second.

M

For cast iron, allowing a factor of safety, the safe stress may be about 1000 lbs. per sq. inch. whence

$$h_1 = \sqrt{\frac{12{\cdot}6\ \text{H.P.}.\,l_1}{\text{DL}n\,.\,n_s}}\ \text{approx.}$$

On account of the fact that the pull is exerted under the pole-faces chiefly, and not uniformly around the armature, and also because of possible sudden strains, the value of h_1 is usually considerably greater than that given by this formula, as is clearly seen in the illustrations.

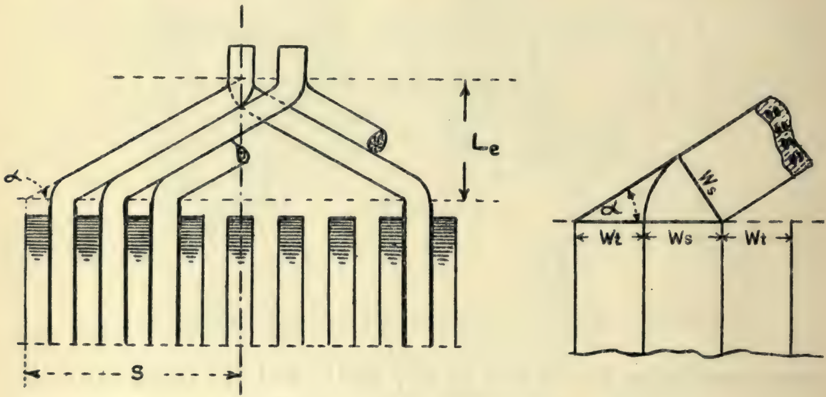


FIG. 109.—END CONNECTIONS.

Armature End Connections.—In the cylinder-wound or "barrel-wound" armature, which is now almost universal, the armature coils project from the armature at either end, and there lie along the surface of a cylinder concentric with the armature. Usually, for various reasons, this cylinder is slightly conical, as is seen in the several illustrations. Part of the surface is developed in Fig. 109, and the construction there shown determines the length, "L_e," that the end-connections must project, and therefore also the length of the end-support. If W_s be the width of a slot, W_t the width of a tooth, then it is seen that—

$$\cos a = \frac{W_s}{W_t + W_s}$$

$$\text{Also } \frac{L_e}{S} = \tan a = \frac{W_s}{\sqrt{(W_t + W_s)^2 - W_s^2}}$$

whence L_e is determined if S be known.

Now, S is half the number of slot-pitches spanned by one coil plus $\frac{1}{2}$ a slot-pitch (nearly), and this is known from the winding scheme.

The total amount that the end windings project, is L_e plus an allowance at either end, as shown in the sketch, and usually $L_e + \frac{3}{4}''$ is a sufficient total length.

Example.—Assume that the armature winding calculated on p. 114 lies on an armature 13 inches diameter with slots 1·4 inch deep and 0·5 inch wide; then the slot pitch at the bottom of the slots is 0·78 inch. The slot winding-pitch from p. 114 is 10 slots = 7·8 inches.

$$\therefore S = 3{\cdot}9'' + 0{\cdot}39'' = 4{\cdot}3'' \text{ nearly}$$

$$\text{Thus } L_e = S\frac{0{\cdot}5''}{\sqrt{(0{\cdot}78)^2 - 0{\cdot}25}} = 4{\cdot}3\frac{0{\cdot}5}{0{\cdot}598} = 3{\cdot}6''$$

$$L_e + \tfrac{3}{4}'' = 4{\cdot}35''$$

Fixing of Armature Coils.—Modern armatures, being provided with slots in which the armature coils lie, have this advantage over the older smooth-core machines—that the pull between the armature and the magnetic field falls on the armature teeth instead of upon the conductors themselves. There is in consequence only one force practically to reckon with in fixing the armature coils, viz. centrifugal force.

The methods in use to guard against movement of the coils are two, viz. wooden wedges fixed in the slots, with binding wires over the end connections; or binding wires only.

When the first method is adopted the teeth are specially shaped to accommodate the strips of wood, as shown in Fig. 89 (2), p. 145.

These strips are often of seasoned and varnished maple; but oak, hornbeam, and ash have sometimes been used instead. The thickness adopted varies, according to the size of the machine, between the limits 0·1 inch and 0·2 inch.

Binding wires are usually made of phosphor-bronze or steel. They are grouped into bands, about $\frac{3}{4}$ inch wide. Where such bands are used on the core, a channel is made around the core by inserting a sufficient number of groups of core-discs of a diameter slightly smaller than that of the armature. The depth of this channel is usually $\frac{1}{16}$ inch or slightly less. Under the binding wires is laid a thin ribbon of press-spahn and of mica, and the individual wires are at intervals soldered together and clamped by small sheet brass or copper clips.

The size of binding wire can be calculated from the strength of the wire, and the centrifugal force to be provided against. The

formula is directly derived from the ordinary centrifugal force expressions, and may be written for convenience—

$$\left.\begin{array}{l}\text{Total sectional area of the}\\ \quad\text{wires in all the bands}\end{array}\right\} = \left\{\begin{array}{l}\text{constant} \times \text{total weight of armature}\\ \quad\text{coils} \times Dn^2\end{array}\right.$$

If D be in inches, and n in revolutions per second, the constant for steel or phosphor-bronze has a value of about 10^{-6}, which allows a stress of 8000 lbs. per sq. inch in the wire.

Example.—A ten-horsepower motor has an armature 11 inches diameter which runs at a speed of 750 r.p.m. The armature is wound with 369 turns of number 13 d.c.c., and the length of one armature turn is 35 inches. Calculate the binding wires.

No. 13 d.c.c. has a weight of about 80 lbs. per 1000 yds.

$$\text{Hence total weight of armature winding} = \frac{369 \times 35 \times 80}{36 \times 1000} = 29 \text{ lbs.}$$

$$\text{Total area of binding wires} = 10^{-6} \times 29 \times 11 \times \frac{750}{60} \times \frac{750}{60}$$

$$= 0{\cdot}05 \text{ sq. inch}$$

If there are five bands of twenty turns each, the area of one wire is 0·0005 inch, corresponding to No. 22 S.W.G. approximately, and the width of each band will be $\frac{5}{8}$ inch about.

Commutator Construction.—The general arrangement and details of various commutators are shown in Figs. 110, 111, and 112.

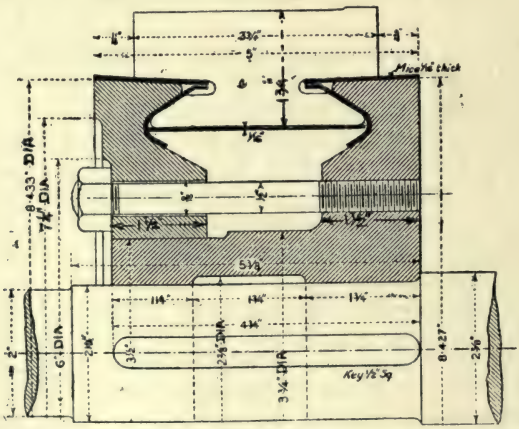


Fig. 111.—Half-section through Small Commutator (Thomson-Houston Co.)

The main points to be observed are—

(1) That the commutator can be removed from the shaft without disturbing either armature-winding or commutator-segments, except in so far as the connections between them are concerned.

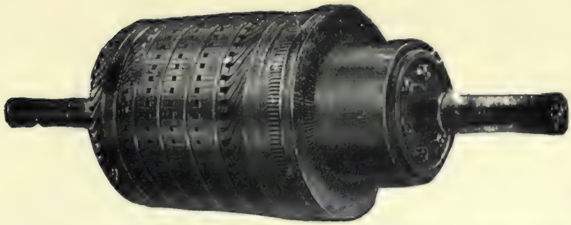


FIG. 110.—CYLINDER-WOUND ARMATURE WITH COMMUTATOR (GENERAL ELECTRIC CO.).

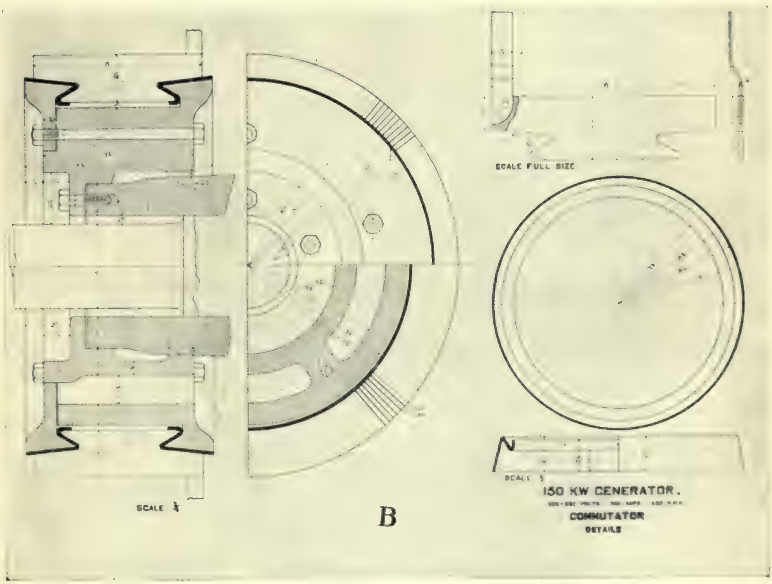


FIG. 112.—DETAILS OF A COMMUTATOR (CRAMP).

[*To face p.* 164.

(2) That in the case of the large machine the commutator is *positively driven* from the armature hub.

Details of construction, showing the usual angles adopted, together with the method of fixing the *risers* for connection to the armature, are shown in Figs. 113 and 114. These risers are usually made from sheet copper (about No. 20 gauge), and they are fixed by rivets and solder to the segments, which are milled away to receive them as shown by the dotted line in Fig. 114, the armature and commutator when fixed appearing as in Fig. 110.

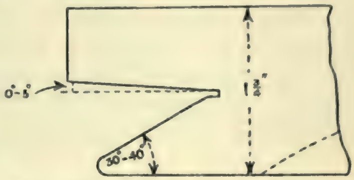


FIG. 113.—COMMUTATOR SEGMENT.

A marked feature of modern commutators, well seen in the illustrations, is the care taken to ensure as much ventilation as possible. The passages through the commutator-bush are not provided solely for cooling the commutator, though they certainly serve this purpose; but they also admit of a current of air right into the armature itself. The air can thus be drawn in at both ends of the armature and expelled

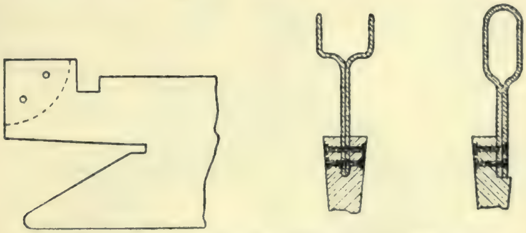


FIG. 114.—COMMUTATOR SEGMENT AND RISERS.

by contrifugal action through the spaces left by the distance pieces between the stampings. The connections from the armature-winding to the risers are usually well soldered, and in the case of bar-wound armatures they are also riveted.

Brush Gear.—Brushes are either fixed, or they are carried by a rocker arranged so as to be adjustable about the commutator. The former plan is pursued in the case of motors which must be reversible, like those for tramcars and hoists; and in these cases the brushes are fixed along the no-load neutral axis.

Rockers are of two kinds: those which are carried on the bearing-shell, like Figs. 115 and 116; and those carried by the field-frame, as in Figs. 117 and 118. The former are used for machines up tc about 50 K.W., and the latter for the larger sizes.

The turned portion of the bearing upon which such a gear as that

illustrated in Figs. 115 and 116 would fit is marked with the letter A in Figs. 98 and 100. Indeed, Fig. 116 is a detail of the actual gear designed for the machine shown in Fig. 98. The adjustment and fixing of the gear is arranged for in the slot B in Fig. 98, through which passes a set-screw into the hole B of Fig. 116. The gear can thus be adjusted and locked without putting a spanner inside the case, which is to the author a convenience well worth attention. Fig. 118 is a scale drawing of a field magnet frame and brush-rocker of the type illustrated in Fig. 117. The scale of the drawing can be fixed from the knowledge that the armature diameter is 26 inches. The machine is the same as that illustrated in Figs. 92 and 112.

The insulation from the frame of the brush-holders and their connections has already been referred to (p. 147). It is usually

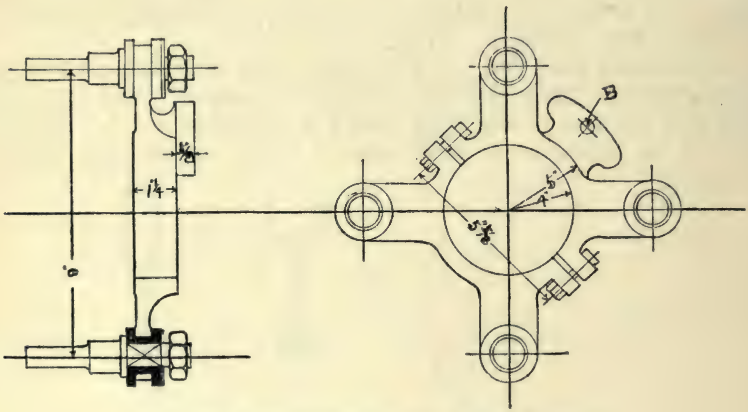


FIG. 116.—DETAILS OF SMALL BRUSH-ROCKER.

carried out by an insulating flanged bush which separates the spindles carrying the brush-holders from the brush-rocker itself, These bushes, as seen in the various illustrations, are usually moulded from ambroin, ebonite, or vulcanite; sometimes in high-voltage machines they are made up from micanite, mica, or porcelain. A dimensioned detail is shown in Fig. 119.

Brush-holders.—The only brushes now used upon continuous-current machines are made of some form of carbon. The brush-holders for these are in general of two forms: (*a*) Box type; (*b*) Lever type. It is quite common for dynamo-makers to buy their brush-holders from firms that have specialized in this line. This Fig. 120 illustrates the brush-holders of Verity's, Ltd., and Table XVI. gives the corresponding standard dimensions. The three upper views in Fig. 120 show the construction of a brush-

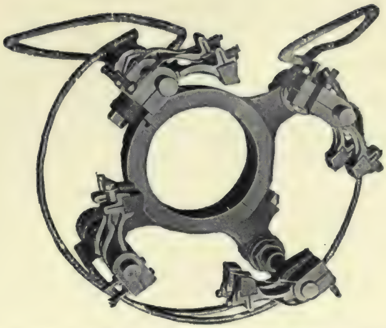


FIG. 115.—BRUSH-GEAR FOR SMALL MACHINE (GENERAL ELECTRIC CO.).

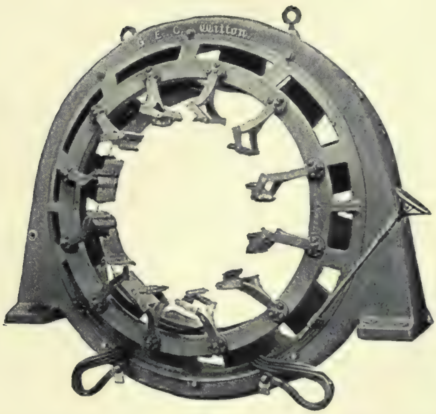


FIG. 117. –BRUSH-GEAR FOR LARGE MACHINE (GENERAL ELECTRIC CO.).

[*To face p.* 166.

holder for very small machines. In such cases a rocker is not used, but the brushes are fixed right on to the machine end-plates. In

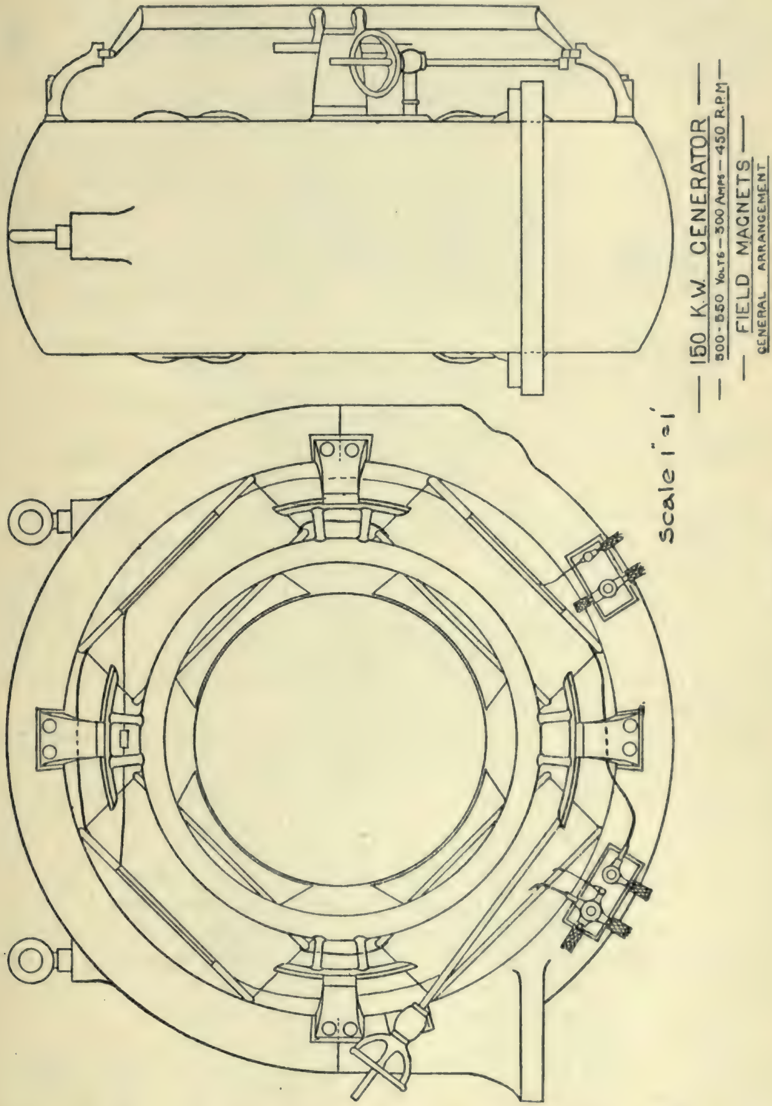


FIG. 118.

this instance, as will be seen from the illustration, the case is moulded from an insulating material, and contains a rectangular

brass tube in which the brush slides. The spring is arranged in the space above the tube, and is of the spiral type. One end is fixed inside the case, and the other is connected to a brass rod in which the carbon brush is firmly embedded. A flexible lead carries the current from the carbon socket to the brush lead terminal. No current passes through the spring.

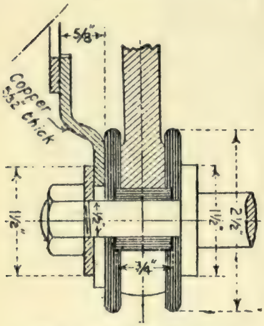


FIG. 119.—DETAIL OF BRUSH-ROCKER INSULATION.

The desiderata of a really good brush-holder for large machines are—

(1) The moving parts should be light, so that the brush can easily follow any small irregularities of the commutator surface.

(2) The brush should be automatically adjustable for wear, and should feed practically radially.

(3) It must be possible easily and quickly to adjust the brush pressure.

(4) The brush should possess a good electrical connection.

(5) It should be possible to change the brush quickly.

These points are most easily met, so the author thinks, by a brush-holder of the box type, *i.e.* of a holder which consists of a box in which the carbon brush slides as it is fed forward by a spring. Such is illustrated in the middle of Fig. 120, the fourth essential being usually provided for by the use of a copper "pig-tail," or flexible connection, one end of which is fixed to the brush, and the other end to the holder. Dimensions of this type of holder are given in Fig. 121 and Table XVI.

Brush-holders of the lever pattern to fulfil these requirements are somewhat complicated, and must be made in thin metal or in aluminium for condition (1). A construction is clearly indicated in the last holder shown in Figs. 120 and 121; and the wavy line in Fig. 121 shows the copper ribbon used to unite the moving and fixed portions. An advantage of the lever type, which does not apply to the box type, is in the firmness with which the brush is held; for slight movements of the brush which tend to set up sparking and rattling are frequently met with. Swelling of the brush with heat, and consequent binding in the box, is a trouble sometimes met with in brush-holders of the box-type.

Another slight advantage of the lever type is to be seen in Table XVI., where one size of brush only is used for all machines, while for the box-type, the Admiralty have no less than five standard sizes.

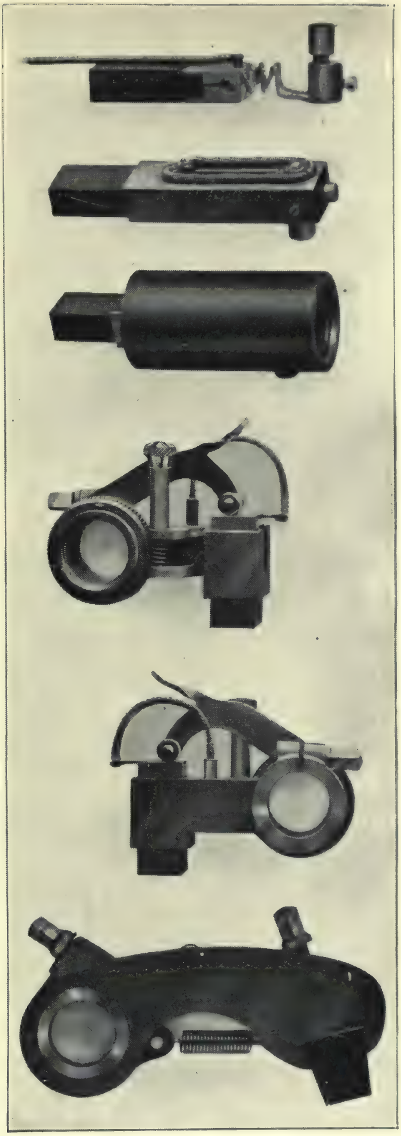


FIG. 120.—STANDARD BRUSH-HOLDERS (VERITY'S, LTD.).

[*To face p.* 168.

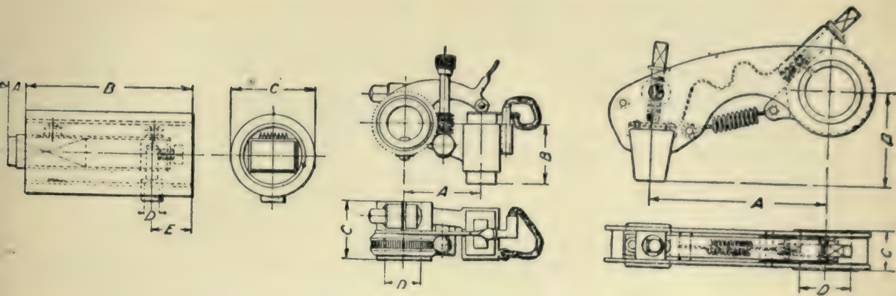


FIG. 121.—DETAILS OF VERITY BRUSH-HOLDERS.

TABLE XVI.

Carbons.	Holder Dimensions.			
	A.	B.	C.	D.
Box Type. (Admiralty sizes.)				
$\frac{5}{16}'' \times \frac{5}{8}'' \times 1\frac{1}{4}''$	$1\frac{5}{16}''$	$1\frac{1}{8}''$	$\frac{15}{16}''$	$\frac{5}{8}''$
$\frac{1}{2}'' \times \frac{3}{4}'' \times 1\frac{1}{4}''$	$1\frac{1}{4}''$	$1\frac{1}{8}''$	$\frac{15}{16}''$	$\frac{5}{8}''$
$\frac{5}{8}'' \times \frac{7}{8}'' \times 1\frac{1}{2}''$	$1\frac{7}{8}''$	$\frac{7}{8}''$	$1\frac{1}{8}''$	$\frac{7}{8}''$
$\frac{1}{2}'' \times 1'' \times 1\frac{1}{2}''$	$1\frac{7}{8}''$	$\frac{7}{8}''$	$1\frac{1}{8}''$	$\frac{7}{8}''$
$\frac{7}{8}'' \times 1\frac{1}{2}'' \times 1\frac{1}{2}''$	$1\frac{7}{8}''$	$\frac{7}{8}''$	$1\frac{1}{8}''$	$\frac{7}{8}''$
Lever Type.				
$\frac{1}{2}'' \times \frac{3}{4}'' \times \frac{7}{8}''$	$3''$	$1\frac{5}{8}''$	$\frac{11}{16}''$	$\frac{7}{8}''$

The sketch of mechanical details here given is necessarily very incomplete. More may be seen of such matters by a few days in a works than by weeks of study of illustrations. But failing the former opportunity, much may be learnt by carefully drawing out details from actual machines, or from the various published sheets of drawings, as for instance those by T. and T. G. Jones.* Several illustrations in the present chapter have been taken by permission direct from the last-named authors, as it is hard to find examples more suitable or better drawn.

* "Machine Drawing," Book 4, Section 1. T. and T. G. Jones. London and Manchester: John Heywood.

CHAPTER XII

COSTS

It has been pointed out, and there is much truth in the statement, that estimating the cost of machines has been too much divorced from designing, and that the designer has not sufficiently considered the cost of his final arrangement. However this might have been in the past, it is certain that present competition compels the strictest attention to the cost of every detail; and each designer should have some ready method of estimating the costs of his designs for comparative purposes, so that he may only put forward for manufacture those which combine economy with efficiency. The expression "ready method" is used advisedly, for no designer should attempt to displace the estimating department: on the contrary, it should be the duty of the estimating department to supply the designer with such figures as shall enable him to carry out his relative costs with fair accuracy.

On account of the vast differences existing between works in size and equipment, it is hardly possible to say in which direction economy must always lie; for the author has seen processes carried out in some works, which in others would be entirely inadmissible on the score of expense. In consequence, the most that can be done is to indicate a proceeding which, if properly carried out in each individual works, will lead to an efficient system of relative costing for different designs.

Whatever system of time-sheets, store-cards, etc., may be adopted, the costs of any machine are finally brought together on to a summary sheet which varies but little in even very different works.

Such a summary sheet is, in its more elaborate form, usually divided into three main portions, viz. *Labour, Material, Miscellaneous.* Each of these is again subdivided.

Under **Labour** is found—

1. **Machinemen.**—To the cost of the labour under this head is added a percentage to cover supervision, and establishment charges such as rent, rates, taxes, lighting, heating, and interest and

depreciation on the cost of tools. This percentage varies from 40 to 120 per cent. according to the proportion which the items included bear to the works turnover. It can only be determined by the auditors at the annual audit, and no two works are quite alike.

2. **Bench-hands**, such as joiners, fitters, pattern-makers, smiths, hand-winders, etc. To the cost of labour under this head must be added a percentage similar to that above, but smaller in value because the ratio of wages per hour to capital costs in tools, etc., involved, is much greater. Here again the proper proportion to be added must be determined by the auditors, and usually ranges from 40 per cent. to 70 per cent.

Under **Material** is found—

Castings, rough material from stores, finished material from stores (bolts, screws, wire, insulating tubes, etc.), timber, etc. To all material bought in this way is added a small percentage to cover the cost of handling, usually about 5 per cent. Some firms add 7½ per cent.

Under **Miscellaneous** is found testing, special fittings, and special expenses not included under labour or material.

To the cost of the article as determined by the sum of the above items (called the *total works-cost*) two more percentages must be added before the selling price is arrived at. One of these, which varies enormously in different cases, is intended to cover selling expenses, such as advertisement, agents, commissions, branch-office costs, etc., and the other is profit.

Calculation of Total Works-Cost.—From summary sheets drawn up on the above lines it is possible to deduce a further summary, in which, for each special article, the relationship of labour and material to material can be ascertained. This gives the total works-cost of the article when once the material involved is known. Now, the latter bears a fairly constant ratio to the cost of "effective material"; *i.e.* to the total cost of all the iron and copper involved in the magnetic and electric circuits of the machine. Thus, if these be calculated, the total works-cost can also be estimated. It is seen that all that is required to enable the designer to compare costs of different designs is the ability to calculate the effective material and its cost, and a knowledge of the connection between this material and the labour expended upon it as given by the summary sheets above referred to.

Some estimators use a much more simple method for determining the selling price. Thus they may add all the labour-costs plus a percentage, and all the material-costs plus 5 per cent., and to the sum of these add a percentage, say 10 per cent., for profit. In this case the establishment and selling charges all appear in the one percentage on labour, which then mounts up to sometimes

200 per cent., on to which is added the cost of material + 5 per cent., and to the total the percentage for profit.

The percentages above mentioned, whilst they no doubt bear excellent testimony to the reduction of labour-costs under modern competitive industrial conditions, yet testify also to the extraordinary cost of persuading a customer to take the article when once it has been manufactured. Thus keen competition, whilst tending towards efficient production, leads also to extravagant methods of distribution and exchange; from which it would seem that a reasonable reduction in competition might, if properly controlled, admit either of an increase in labour-payments on the one hand, or of a substantial advantage to the consumer on the other.

Examples of Costing.—To illustrate the methods of costing above set forth consider the following case:—

The cost-cards for the armature and shaft of a 250 K.W. 500-volt moderate-speed generator yield the following summary:—

	£	s.	d.	£	s.	d.	£	s.	d.
Labour.									
Machinemen . . .	6	0	0						
100 per cent. . . .	6	0	0						
				12	0	0			
Bench hands . . .	3	0	0						
Winders	13	0	0						
	16	0	0						
50 per cent. . . .	8	0	0						
				24	0	0			
Total labour							36	0	0
Material.									
Steel shaft . . .	9	0	0						
Laminations . . .	40	0	0						
Copper	20	0	0						
Insulating material . .	7	0	0						
Spider	4	0	0						
Nuts, bolts, binding-wire, and miscellaneous .	10	0	0						
	90	0	0						
5 per cent. . . .	4	10	0						
Total material costs							94	10	0
Total works cost of armature							130	10	0

In like manner, from similar summaries of the other parts of the same machine the following figures are obtained:—

	£	s.	d.
Total works cost of magnets, yoke, bearings and field coils	250	0	0
Total works cost of commutator and brush gear .	160	0	0
Total works cost of whole machine . . .	540	10	0

Now suppose that the designers in the works where the above machine is made are asked to get out designs for 500-volt machines of other sizes. Suppose, also, that they have access to the summary sheets mentioned above. Then from these they take first the cost of the effective material as follows:—

	£	s.	d.	£	s.	d.
Iron—						
Yoke	30	0	0			
Poles and shoes	24	0	0			
Armature core	40	0	0			
				94	0	0
Copper—						
Armature	20	0	0			
Shunt-field	50	0	0			
Compounding	8	0	0			
Commutator	45	0	0			
				123	0	0
Total cost of effective material . . .				217	0	0

$$\text{Ratio } \frac{\text{total works cost}}{\text{cost of effective material}} = \frac{540}{217} = 2{\cdot}46$$

If other sizes of machines can be similarly analyzed, a rough curve can be plotted between the above ratio and the kilowatts output per revolution per minute, from which approximate values for the ratio in the case of the machines to be designed can be obtained. If great care is taken in reducing the machining as well as the effective material, a change in the ratio may be found when the first machine goes through the shops, and this change may be allowed for in subsequent designs, so that a better approximation is obtained at each stage, new curves being finally constructed. In any case, so long as even approximate values for the above ratio are known, the designer can compare his preliminary trial designs with the selling prices of similar machines by other makers, until he arrive at a machine which, for the same price, has some advantages over that of his competitors.

Values of Cost-Ratios.—Where no such ratio can be obtained,

the following, taken from the practice of a certain works, may be used as a rough guide or check:—

TABLE XVII.

K.W. output.	R.P.M.	Ratio $\frac{\text{total works cost}}{\text{cost of effective materal}}$.
500	250	2
300	300	2·35
200	400	2·5
100	450	2·8
80	500	3
60	600	3·2
40	700	3–4 *
20	800	
10	800	

The following, for another works, give values for short-rated totally enclosed series crane motors:—

TABLE XVIII.

B.H.P. per rev. per min.	Ratio $\frac{\text{total works cost}}{\text{cost of effective material}}$.
0·01	3
0·02	2·8
0·04	2·4
0·06	2
0·08	1·9
0·12	1·8
0·16	1·6

Costs of Effective Material.—In calculating the cost of the effective material, the parts considered are those given in the example on p. 173. The weights of these parts are easily deduced from the design, and at the present time the following prices are representative in England:—

* These values vary enormously, according to the number of machines of one size put through at one time.

Iron and steel—		
Armature stampings . . .	4*d*. to 5*d*.	per lb.
Pole-face stampings . . .	2½*d*.	"
Iron castings	¾*d*. " 1¼*d*.	"
Steel castings	2*d*. " 2½*d*.	"
Mild-steel and wrought-iron bar, etc.	1½*d*. " 1¼*d*.	"
Copper—		
Wire for magnets and small armatures	9*d*. " 1/-	"
Strap for armatures and series-windings	9*d*. " 10*d*.	"
Commutator-sections . . .	11*d*. " 1/-	"

Naturally all these prices will vary with the market for the material; the small variations shown are due in the case of castings to the cost of moulding, or to the difficulty of the pattern; and in the case of wire to the size of the wire, *i.e.* to the labour entailed in drawing. In the case of stampings, the cost of the tools and the waste material have to be considered. Armature stampings are usually No. 26 gauge, and pole-face stampings No. 20 or 21. Examples of the cost of effective material are given on pp. 188, 196, and 210.

Other Methods of Costing.—Although the foregoing may seem only a rough and ready method, yet still simpler plans have been proposed and used. Thus Hobart suggests two formulæ with constants which are exceedingly simple. In the first, he adds to the cost of the effective material a sum which he calls the non-effective cost, and which may be expressed in the form—

$$(\text{constant})D_1^2 + (\text{constant})D_1L$$

where D_1 is the diameter outside the yoke. D_1 and L are in centimetres. These constants, of course, must be derived for each works, but in one instance they are quoted as 0·1 and 0·14 respectively.*

His second method is still more brief, for he writes it: (constant) $\times$ DL_1, where L_1 is the length of the armature over the end connections, and consequently may be written roughly as *—

$$L_1 = \frac{3\pi D}{4 \text{ poles}} + L$$

Such methods can only be relied upon when a large number of machines enable the constants to be very closely determined. Other methods have been suggested by E. K. Scott, Mavor, Wilson, and others.†

Cost of Component Parts.—It is generally impossible to form an estimate of the cost of the field-magnet or armature until the preliminary design is fairly complete. With the commutator,

* H. M. Hobart, *Electrician*, vol. 51, p. 850.

† See *Jour. Inst. E.E.*, p. 400, 1893; and p. 160, 1897.

however, this is not the case, as its cost depends chiefly upon the current to be collected.

Cost of Commutators.—It has been already pointed out that the radial depth of commutator copper varies but little, even over a wide range of sizes. Perhaps it may most conveniently for present purposes be taken at 2″. It is then clear that the cost of the commutator is almost independent of any factors other than current to be collected and peripheral speed. Thus, if we assume an average value of 30 amps. per square inch for the current-density under the brush, the total commutator losses may be put into terms of the current to be collected and the peripheral speed (V_c). From p. 133 we obtain—

Electrical losses + friction losses = total current $(1{\cdot}8 + 0{\cdot}000678V_c)$

$$\left.\begin{array}{l}\text{Area of necessary cylindrical}\\ \quad\text{radiating surface (from p. 86)}\\ \quad\text{allowing 4 watts per sq. inch}\end{array}\right\} = \text{total current}\,(0{\cdot}45 + 0{\cdot}00017V_c)$$

This radiating surface multiplied by the average depth of say 2″ gives a first approximation to the volume of active copper in the commutator, so that the cost at 1/- per lb. will be—

$$\text{Total current}\,(0{\cdot}27 + 0{\cdot}0001V_c)\ \text{shillings}$$

which corresponds to about 1·8*d.* for each watt dissipated. This value usually lies between 1·2*d.* and 2½*d.* according to the depth of segment and the watts per square inch allowed. The cost of the commutator increasing, as shown, with increase of current, tends to make low-voltage machines cost more than high-voltage machines of similar output. This, however, is often offset by decreased insulation and winding costs, so that as a general rule there is little difference in cost between 200- and 500-volt machines. Outside these limits the relative cost depends upon the size of the machine; for in small machines the number of commutator sizes stocked, the available length of frame, etc., will obviously affect the range of voltages over which the design can be used with economy. Larger machines, on the other hand, can be considerably altered in detail to suit special outputs without greatly increasing the cost of manufacture, because, as has been seen, the ratio of total works-cost to cost of material is so much smaller.

General Effects of Design Ratios upon Cost.—One of the best indications of excellence of design as regards cost is the ratio ampere-turns per pole × number of poles divided by total armature ampere-conductors. The values of this ratio, which correspond to minimum cost, have already been referred to on p. 62, but the figures there given depend very much upon the type of machine, and can rarely be approached except when interpoles are employed. For traction generators without interpoles the ratio will often lie between 600 and 800, because of the large number of commutator sections

necessitated by a lower value of the commutation constants. In the comparative calculation carried out on pp. 189, 196, the effect of commutation constants upon the ratio is clearly seen, and, generally speaking, so much depends on the purpose for which the machine is intended, on its voltage, speed, and rating, that it is impossible to lay down hard-and-fast lines without a very definite specification to work to. One may say, however, with comparative certainty, that the interpole machine with lowest total works-cost does lie somewhere within the limits named on p. 62; and for machines without interpoles the following table is typical of good modern practice :—

TABLE XIX.

NON-INTERPOLE MACHINES.

K.W. output.	Value of ratio $\frac{Y}{X}$.	
	Low speed.	High speed.
10	550	650
50	500	750
100	500	850
250	600	950
500	800	1100
1000	850	1200
1500	900	1200

In considering this ratio it must always be remembered that few designers are called upon to consider a frame for one single output unless it be for a large and special machine. A machine of given core-length for a certain speed will have that core-length increased in almost inverse proportion for lower and higher speeds, so that the above factor will be changed according to that speed, and it is from this point of view that designing from such a ratio is most unsatisfactory, and would be entirely out of the question were it not for the comparatively small variation between small and large machines. It must not be supposed that the limits above given by any means represent universal practice; indeed, the author could quote cases in which for 150 K.W. machines the ratio was as high as 1200, but he does not consider that such designs could be very economical.

Generally speaking, it may be said that increasing the flux per pole increases the constant losses, and, up to a certain limit, also the

costs. It has the effect, too, of lowering the efficiency at low ratings, and so unfits the machine for work when totally enclosed. It is true that it tends to raise the efficiency somewhat at high ratings, but the gain in this respect does not counterbalance the disadvantages mentioned.

A small flux per pole is therefore desirable for most purposes, and especially so where the machine must be sometimes totally enclosed. The comparative value of the flux is obviously measured by the ratio dealt with above, and the possibility of decreasing it is checked in non-interpole machines by the great increase in the armature diameter, and in interpole machines by the gradually increasing cost of the interpoles.

Other points which influence the cost are: (1) the rating of the machine, and the amount of ventilation; (2) the voltage of the machine, and consequent possible space factor; (3) the speed of the machine.

The first of these has been dealt with under Temperature Rise (Chaps. VI. and VII.) and under Division of Losses (Chap. III.). We may here add that when the machine is totally enclosed and short-rated, the mass of the frame often has more to do with the temperature rise than has the radiating surface. It is impossible to properly design short-rated motors until some agreement is reached as to the tests which they should be called upon to fulfil; an example dealing with such a machine is given on p. 213.*

Often a designer will find himself confronted with the relative effect upon the cost of two dependent variables; thus reducing the section of a yoke will increase the amount of field copper, but decrease the weight of steel or iron required, and the question is, What section of yoke should be adopted? The point can often be decided algebraically by obtaining an equation connecting the two quantities, and finding the minimum of the sum of the two. More often the expression is too complicated to be of any use, and then a graph of the two quantities plotted upon the same sheet may be used for determining the sum of the two. Thus Fig. 122 gives the cost of steel and copper respectively, plotted against the density in the yoke in lines per square inch for a 60 K.W. machine at 460 R.P.M. The economical density in this case is about 87,000 lines per square inch, and as this is below saturation it is a possible density. By such means the cost of existing machines may frequently be considerably reduced.

We may conclude by collecting a few of the more important considerations affecting cost, and, as usual, these will be put under the headings of Small Machines and of Large Machines.

* See a recent paper by Dr. Pohl, *Electrician*, March 25, 1910.

(*a*) *Small Machines.*—For each part the very best material of its kind should be specified, and the amount of machining should be reduced to a minimum, except in those cases where absence of machining would entail extra hand labour; as, for instance, in the case of a shaft, which it is often more economical to turn out of a large bar, leaving the necessary collars, than to make approximately to shape as a forging to be afterwards turned.

Remember the necessity of flexibility of design, especially as regards the length of armature and commutator for high and low voltages (p. 33). Remember, also, that variations in speed can be largely allowed for by the method of lengthening and shortening the armature, and in the case of very high speeds, by the addition of interpoles.

Very small machines in which it does not pay to vary armature

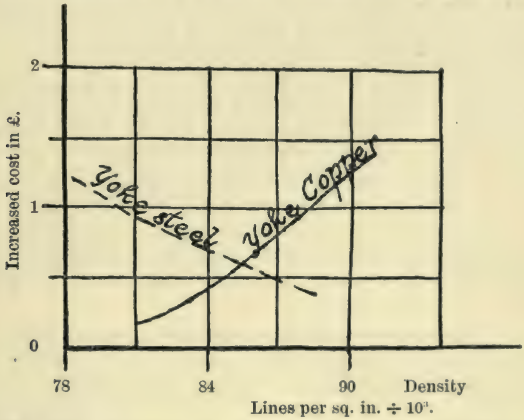


Fig. 122.—METHOD OF DECIDING BEST YOKE-SECTION.

and commutator lengths most usually have their output reduced as the voltage is raised, and slight advantages may sometimes be gained by changes in the ampere-turns per pole and in the efficiency. Naturally also in these cases the output will change with the speed.*

For appearance' sake some makers will put the pole nearer to one end of the yoke, as this has the effect of causing the commutator to stand out less; such an arrangement, however, renders the machine impossible as a rotary converter or double-commutator machine without a fresh pattern for the end-plate.

(*b*) *Large Machines*, being much less dependent upon labour cost, can be varied within wider limits. Thus the number of poles may

* For a full discussion of these points, see *Electrial Engineering*, vol. i. p. 52, January 10, 1907.

be increased as the current increases, but otherwise the advantages obtained by altering the armature and commutator lengths are similar to those mentioned under (*a*). Flexibility is, however, less important, and distribution of material more important. Consequently it is necessary, after working out a preliminary design with satisfactory proportions, to vary these by many subsequent trial designs, until the minimum cost that is consistent with good performance is obtained.

Tendency of Modern Manufacture.—The necessity for sparing no effort to arrive at the very best possible compromise between performance and cost has been frequently emphasized in the preceding pages. No text-book, however complete and however well conned, can possibly enable a designer to dispense altogether with the experience necessary for successful competition, chiefly because of the different conditions existing in different works. But should the truth of this statement be yet held in doubt, eloquent testimony is to be found in a bald comparison of the conditions existing only a few years back with those which obtain to-day (1910). Thus the following table, though it refers to one size only, is typical of what has taken place throughout the whole of the industry:—

TABLE XX.

COMPARISON OF COSTS AND PRICES FOR A SEMI-ENCLOSED CONTINUOUS-CURRENT MOTOR OF 7 B.H.P. AT 750 REVS. PER MIN. SUITABLE FOR 460 VOLTS.

Year.	Cost of Material, etc.	Cost of Labour, etc.	Total Works-cost.	Selling Price.	Profit per cent. of Works-cost.
	£	£	£	£	Per cent.
1902	22	22	44	56	27
1910	15	10	25	30	20

Thus in eight years (for this size) has the material been reduced by nearly 32 per cent., the labour charges by 55 per cent., the total works-cost by 43 per cent., and the profit by 58 per cent. And yet the efficiency is at least as high as before, and wages have risen rather than decreased. The tendency thus manifest is still to be traced in catalogues, tenders, and balance-sheets.

EXAMPLES OF PROCEDURE IN DESIGN

The object of the examples which follow is to illustrate the bearing of the many considerations discussed in the preceding chapters. It should be very apparent that the design for a dynamo or motor can at best only be a good compromise; and that the very data available are, even for machines of the same output, dependent upon factors of a very indeterminate nature, such as position of works, price of raw material, size of works, number of machines of one output required, and many other questions. For these reasons it is desirable that, having paid attention to the many conflicting conditions mentioned, each designer should formulate for himself a method of attacking the problems he is called upon to solve. Reliance upon a definite set of formulæ is apt to lead to neglect of the conditions upon which such expressions are based, so that on a return to root principles it may be found that some important ratio involved in the algebra has been lost sight of. In the author's opinion, sets of equations are generally only of value to the man who put them together for his own use.

The illustrations given, therefore, will be such as to indicate (i.) methods of arriving quickly at the outline of a satisfactory design; and (ii.) the directions in which the work on this first outline should proceed in order to arrive at a final form.

It is possible broadly to discriminate between two classes of machine, viz.—

1. Small machines.
2. Large machines.

By small machines we mean such sizes (usually less than 30 K.W.) as are never manufactured one at a time. These must be considered, therefore, from various standpoints, as, for instance, adaptability to various voltages and speeds, possibility of enclosing, and output when enclosed.

By large machines we mean those which may fairly be considered as individual designs.

In regard to the first class, it has already been pointed out that great divergence of opinion exists as to the proportions and densities to be employed, so that in order to commence a design it is often necessary first to find out in which direction modern machines are tending; next, to rough out a trial design; and, finally, to correct this for flexibility of output and cost of production.

In both classes we can discriminate between machines intended for

constant pressure (*i.e.* shunt and compound dynamos and motors) and those intended for varying pressures—usually series wound. The latter are generally ruled by considerations which do not occur with the former, and consequently they will be treated separately.

Constant Pressure Machines. Problem I.—Let it be supposed that a line of small motors is required ranging from 5 to 15 H.P., and that it is proposed first to consider a machine of about 12 H.P., running at 900 r.p.m. (= 15 r.p.s.)

We may first analyze a few designs of such machines by competing makers, such as the three given below—

TABLE XXI.

Details.	A.	B.	C.
H.P.	5	10	5
R.p.m.	600	950	700
Voltage	220	500	110
Type	Semi-enclosed	Semi-enclosed	Totally enclosed
Material of yoke	C.I.	C.S.	C.S.
Field-poles	C.S.	C.S.	C.S.
Armature winding	2-circuit	2-circuit	2-circuit
Armature diameter	9″	$9\frac{3}{4}$″	10″
No. of poles	4	4	4
Gross length of core	$4\frac{3}{4}$″	$6\frac{3}{4}$″	6″
Ventilation	None	One space $\frac{1}{4}$″ wide right through the armature	One space $\frac{5}{8}$″ wide
Slot depth	1″	0·8″	0·625″
No. of slots	31	47	64
Pole-arc length	$5\frac{1}{2}$″	$5\frac{3}{4}$″	5″
Length of air-gap	0·15″	0·125″	0·15″

From such data some general probabilities may be gleaned which will afterwards be of use. For instance, in this case the number of poles to try is evidently 4, and the armature-winding will be best as two-circuit.

The next step may be to decide a few general points from which a number of designs may be quickly outlined. We may take—

Yoke	Cast iron, with a density of about	35,000	lines per sq. in.
Pole	Cast steel, ,, ,, ,,	100,000	,, ,,
Pole-face	Laminated ,, a face density of	45,000	,, ,,
Tooth-roots	Laminated, uncorrected density of	150,000	,, ,,
Armature below teeth, with a density of		85,000	,, ,,

λ assumed 1·25.

Space factor for 440 volts, 0·24 (cf. p. 147)
" " 200 " 0·28 (cf. p. 147)
V_r, with hard brush 2

Section of pole, circular. Ratio $\dfrac{\text{pole arc}}{\text{pole-pitch}} = 0{\cdot}7$
Efficiency at full load, 86 per cent. (cf. p. 27)
Temperature rise, 40° C. $\rho = 78 \times 10^{-8}$

We assume the most important relationships in the design to be—

The ratio $\dfrac{Y}{X}$.
The value D^2L.
The value V_r.
Temperature rise.
Division of losses.

General Relationship in Terms of *d*.—Given the equation on p. 128, any one of the above relationships can be put in terms of the pole-diameter, or the armature diameter. This the reader can carry out for himself, if he wishes to; we have already given reasons for not doing it. He might find, however, values somewhat as follows :—

$$Y = \pi d^2 \times 10^5 \qquad V_r = \frac{14 \text{ t.p.s. watts}}{d 10^5}$$

$$D^2L = 6{\cdot}45 d^3 \qquad \text{Copper loss in armature} = \frac{{\cdot}00288 \times \text{watts} \times X}{nd^3}$$

$$\text{Iron losses} = 3{\cdot}1 d^3 \qquad X = \frac{EC \,.\, 60}{\pi d^2}$$

and so on. The value of these is in some cases considerable. Thus, see how quickly the ratio $\dfrac{Y}{X}$ will change as regards the pole diameter.

Division of Voltage and Current.—As suggested on p. 33, we may, if necessary, adjust the length of armature-core and commutator to suit the voltage and current. We will therefore commence the design for a medium voltage, such as 200 to 250.

The output then is about 8950 watts ; input = 10,400 watts.

Total current = say 41·6 amps. @ 250 volts.

The flux per pole × number of poles is evidently $\pi d^2 \times 10^5$, where d is the pole diameter.

The active wires (w) on the armature for a two-circuit winding will be, from the E.M.F. formula—

$$w = \frac{10^8 \,.\, 60 \,.\, E \,.\, 4}{2 \,.\, 900 \,.\, \pi d^2 \,.\, 10^5}$$

The current per wire is $\dfrac{C}{2}$.

Thus—

$$\frac{Cw}{2} = X = EC \cdot \frac{400}{6\pi d^2}$$

and

$$\frac{Y}{X} = \frac{6\pi^2 d^4 \cdot 10^5}{400EC}$$

which, as being independent of current and voltage, is a very useful expression.

In the present instance EC is the armature output, which will be the total input minus the copper loss and the field loss. The latter we do not know, but we may for present purposes take a value from Fig. 20, say about 3 per cent. or 300 watts. Afterwards this may be modified.

Again, from the curve Fig. 21 we see that the armature copper loss will be about $3\frac{1}{2}$ per cent., so that the corresponding voltage drop is $\frac{3 \cdot 5}{100} \times 250 = 8 \cdot 8$ volts; and the voltage drop at the brushes (Fig. 80) $= 2 \cdot 4$ volts total. Therefore the E.M.F. to be generated by the armature at full load $= 250 - (8 \cdot 8 + 2 \cdot 4) =$ say 238 volts, and EC = about 9700 watts.

Now, from Table XIX. we select a trial value for $\frac{Y}{X}$.

We know that the value should be as low as is possible, and we therefore decide first to try 550.

Thus inserting in the formula—

$$\frac{Y}{X} = \frac{6\pi^2 d^4 . 10^5}{400EC}$$

the values for $\frac{Y}{X}$ and EC, we get $d = 4 \cdot 4''$, or say :—

(1) $d = 4\frac{1}{2}''$
(2) Area of pole core section = 16 sq. ins.
(3) Flux per pole $= 16 \times 10^5$
(4) Flux issuing from shoe $= 12 \cdot 8 \times 10^5$ lines
(5) Area of shoe face $= \frac{12 \cdot 8 \times 10^5}{45{,}000} = 28 \cdot 5$ sq. ins.
(6) Axial length of shoe $= L = 4\frac{1}{2}''$ say (cf. p. 20)
(7) Pole-arc $= 6 \cdot 3''$
(8) Armature diameter $= \frac{4 \times 6 \cdot 3}{0 \cdot 7\pi} = 11\frac{1}{2}''$
(9) Ratio $\frac{p \cdot d}{D} = 1 \cdot 56$ (cf. p. 20)
(10) Total armature conductors (2-circuit) = 620
(11) Amp. wires per inch periphery = 355
(12) $D^2L = 596$

In the formula on p. 15, adopting $m_2 = 1 \cdot 1$, $m_1 = 1$, then—

$$K = 0 \cdot 34$$

(13) Depth of slot $= \cdot 08D = 0 \cdot 92''$ *

* Note that this may sometimes be increased by $\frac{1}{32}''$, as on p. 191.

Aggregate area of all the slots = $\frac{1}{2}$ armature circumference $\times$ 0·92
= 16·6 sq. ins.

(14) Aggregate copper area = $16{\cdot}6 \times 0{\cdot}28 = 4{\cdot}65$ sq. ins.

(15) Area of one conductor $= \dfrac{4{\cdot}65}{620} = {\cdot}0075$ sq. in.

Current density $= \dfrac{20}{0{\cdot}0075} = 2700$ amps. per sq. in.

(16) Length of mean turn $= 2 \times 3{\cdot}6'' + 3\pi \dfrac{D}{4} = 34''$

(17) Resistance of armature at 60° C. = 0·274 ohm

(18) Armature copper loss = 440 watts approx.

(19) Volts drop = 11

These values (18) and (19) are somewhat higher than those assumed, and may be corrected later.

(20) Armature section below teeth $= \dfrac{1{\cdot}28 \times 10^6}{85{,}000 \times 2} = 7{\cdot}5$ sq. in.

(21) Armature radial depth $= \dfrac{\text{section}}{\text{net length}} = \dfrac{7{\cdot}5}{3{\cdot}6} = 2{\cdot}08''$

(22) Internal diameter of stamping = 5·5″

(23) Weight of stampings neglecting slots = 79 lbs.

(24) Frequency = 30

(25) Iron loss (constant = 1·8) = $1{\cdot}8 \times 30 \times 0{\cdot}085 \times 79$
= 363 watts.

V_r (see p. 128) = 2 = $0{\cdot}38 \times$ turns per section nearly

(26) Turns per section with 4 brush spindles = 5 as a maximum

(27) From the particulars as outlined, the total armature losses will be about 800 watts. Also, since the maximum permissible turns per section without interpoles are five, and these must correspond to the highest voltage for which the machine is to be used (500), we get an idea of the maximum number of slots. The number of conductors at this voltage will be about 1200, and it has been shown that for a two-circuit winding three coils per slot and an odd number of slots are convenient (p. 114). Thus we conclude that about 40 slots would be suitable. If the machine be required for 500 volts and 800 r.p.m., less than 37 slots would introduce difficulties, and to provide for lower speeds still, many makers would adopt 41 slots, and some would have two standard discs with, say, 37 and 41 slots respectively. These are matters for individual judgment, and we shall assume that 37 (which occurs in the list on p. 115) is the number selected.

(28) **Commutator.**—With the higher voltage, then, 111 sections will be required. On the lower voltage half this number will suffice, but it would hardly pay to stock two types of segment for this machine, though some makers do so. Since about 620 conductors are required for 250 volts, we are bound to choose either two or three turns per section with 111 sections, and V_r is only a limiting factor on the higher voltages.

Thus, if 0·2 be the limiting thickness of one section plus its insulation, the minimum diameter of the commutator is 7″, and its peripheral speed is 1650 ft. per minute.

(29) Commutator friction loss (p. 133) $= 0{\cdot}00226 \times 40 \times 1650 \times 0{\cdot}28$
$= 42$ watts

(30) Commutator resistance loss (p. 133) $= 1{\cdot}2 \times 40 \times 2 = 96$ watts

Total 138 watts.

Cylindrical surface, allowing $2\frac{1}{2}$ watts per sq. in. $= 55$ sq. ins.

Commutator length $= 2\frac{1}{2}''$ axially

Check against Brush Surface.—With four arms, contact surface per arm $= \frac{2}{3}$ sq. in. Allowing brush $\frac{1}{2}''$ thick (covers two segments) gives axial length of $1\frac{1}{3}''$; so that the commutator dimensions are ruled by temperature rise, and $2\frac{1}{2}''$ will be sufficient length, unless only two brush-arms be used.

Division of Losses.—We have—

Friction of bearings, etc., say 2% (cf. p. 27)	179	watts
Commutator friction	42	,,
Iron loss	363	,,
Armature resistance loss	440	,,
Commutator ,, ,, ...	96	,,
	1120	,,

The total losses are to be $(10{,}400 - 8950) = 1450$

(31) So that the shunt field loss will be $(1450 - 1120) = 330$ watts

The constant losses then $= 914$

The variable losses are $= 536$

And the ratio $\dfrac{\text{constant}}{\text{variable}} = 1{\cdot}7$

The latter figure is not specially good; it is, however, a fair average (cf. p. 31), and may be improved later.

(32) **Temperature Rise. Armature.**—The total watts to be dissipated by the armature are 800, and we will compare the probable temperature rise as given by methods 1, 2, 3, pp. 81–82.

Method 1.—Length of armature over end connections $= 11\frac{1}{4}''$ (p. 84).

$$A = 11\tfrac{1}{4}'' \times \pi \times 11\tfrac{1}{2}'' = 406 \text{ sq. ins.} \qquad P = 800$$

Now, although the machine is small, the armature, being short and having a ventilating slot, is certainly well ventilated. Hence, we take $a = 40$–45, say 40. Then $V = 2720$ feet per minute, and $T = 36°$.

Method 2.—Allowing for one ventilating space—

$$A = 768,\ a = 85,\ P = 800,$$

we get
$$T = 37{\cdot}5°$$

Method 3.—Fig. 47. The armature should be capable of dissipating about 1000 watts.

So the average of these calculations should prove that the temperature rise will be well on the right side.

Field.—Watts to be dissipated per coil $= \frac{330}{4} = 82\cdot5$.

If we allow in Fig. 44, $d_c = 3''$ and $r_2 = 2\frac{1}{2}''$, and the coil be supposed to be taped and impregnated, we may take C_h as low as 180.

Now, from the equation on p. 75—

$$\begin{aligned} A_m &= 4\cdot7d_c^2 + 9\cdot4r_2(l_c + d_c) + 6\cdot28l_cd_c \\ &= 113 + 42\cdot44l_c \end{aligned}$$

Substituting this in the heating equation (p. 72), we get for a *mean* temperature of 60° C—

$$l_c = 3\cdot2''$$

which is quite reasonable, and corresponds to a pole length of about $3\frac{3}{4}''$.

First Approximation to Field Ampere-turns per pole (cf. p. 42).—

Density at pole-face = 45,000

From Fig. 13 assume length of gap 0·125″.

Then, since pole-arc = 6·3″
Distance between pole-shoes = 2·7″
Ratio Table III. (p. 40) = 21·6, and the constant = 3·35
(33) Effective pole-arc = 6·72″ (actual value of fringing factor = 1·07 instead of 1·1 as assumed)
Effective pole-face area = 30·4 square inches
Density in air-gap if there were no slots = 42,000
Slot width = 0·487

$$\text{Ratio } \frac{a}{b} \text{ in Fig. 26} = 1$$

$$\text{Ratio } \frac{b}{g} \text{ in Fig. 26} = 3\cdot9$$

Corresponding constant = 1·28
(34) Actual air-gap density = 42,000 × 1·28 = 54,000
(35) Air-gap ampere-turns = 0·313 × 54,000 × 0·125 = 2100
Density at tops of teeth = 106,000
Density at roots of teeth = 154,000
Average density = 130,000
(36) Corresponding ampere-turns = 1000 × 0·92 (Fig. 27) = 920

(37) Total ampere-turns for gap and teeth = 3020

(38) **Armature Cross Ampere-turns per pole** $= \dfrac{310 \times 20\cdot5}{4} = 1600$

(39) $$\frac{\text{Ratio ampere-turns for gap and teeth}}{\text{armature cross ampere-turns}} = 1\cdot88$$

If we allow total ampere-turns per pole = ampere-turns for gap and teeth × 1·2, we see that the former will be about 3600. A simple

calculation on the lines of that on p. 76 shows that about 3000 turns of No. 20 S.W.G. will easily go into the space calculated from temperature rise, and will provide the 3600 ampere-turns when expending the required watts at the specified pressure.

(40) **Approximate Weights and Costs of Effective Material.**—

	£
Armature—Iron, 80 lbs. @ 5*d*. . . .	1·67
Copper, 24 lbs. @ 1/- . . .	1·20
Magnets—Poles and shoes, steel, 68 lbs. @ $1\frac{1}{2}$*d*.	0·43
Yoke, cast iron, 420 lbs. @ 1*d*. .	1·75
Coils, 90 lbs. @ 10*d*. . . .	3·75
Commutator—25 lbs. copper @ 1/- . . .	1·25
	£10·00

If we assume that the ratio of total works cost to cost of effective material is for this size $3\frac{1}{2}$, the total works cost of this machine would be about £35. The present (1909) selling price is less than £40, so that the margin is too small.

Criticisms of First Design.—The preceding paragraph shows that the cost of the machine is too high, and we shall now indicate a method of arriving at a better result. It is evident that we must either (1) decrease the cost of production or (2) increase the output for the material used.

Alterations to Reduce the Cost.—The heaviest item in the table of costs is the field magnet copper. This, however, cannot be reduced without either a corresponding reduction in the field ampere-turns, or an increased field-loss. The former is the only practicable course.

The ratio (39) is 1·88, but according to p. 124 it need not much exceed 1·3.

The cheapest way of reducing the field ampere-turns is by shortening the air-gap; thus, with an air-gap of $\frac{3}{32}''$ instead of $\frac{1}{8}''$ the gap-density, after allowing for the various correction factors, becomes about 55,000, the ampere-turns for gap and teeth become 2540, and the total ampere-turns per coil become 3100. Now, equations worked out like those on p. 76 show that a coil smaller than that already arranged will hardly dissipate 82·5 watts.

Three courses are open—

(*a*) Reduce field lost watts and increase armature lost watts.
(*b*) Use a ventilated field-coil.
(*c*) Change the pole-shape and get a larger cooling surface.*

All these alternatives could be tried; we prefer the first, especially as the armature will (from the temperature calculations) dissipate rather

* It should be observed that this very difficulty not infrequently favours the use of a field-pole of rectangular section, besides the fact that a round coil leads to a yoke of rather larger diameter for a given flux.

more than is at present required of it. Thus with field-coil depth of 2″, 3100 turns per coil of No. 21 will, with a field-current of one ampere, give the required ampere-turns, with a loss of 62 watts per coil.

To maintain the same efficiency, then, the armature must now dissipate 522 watts due to resistance, and 363 due to iron; total 885. This it will apparently do, as new temperature calculations show.

The armature current thus becomes 43·6 amperes, and field-current one ampere. Thus—

Total input	= 44·6 × 250 =	11180 watts
Field loss	= 250	
Armature loss	= 885	
Commutator loss	= 138	
Bearings, etc.	= 179	
Total . . .		1452 watts
Output . . .		9728 watts = 13 H.P.
Efficiency	= 87%	

The ratio ampere-turns for gap and teeth ÷ cross ampere-turns of the armature becomes $\frac{2540}{1700} = 1·49$, which is a great improvement on the previous design; and the cost of net effective material is reduced from £10 to £9·25; so that with the same ratio as recently used the total works cost becomes £32·4, leaving a rather better margin considering that the horse-power is now raised.

Without going into the cost of non-effective parts, such as end-plates, bearings, etc. (to do which requires a full knowledge of the particular works), this result is about the best that can be obtained for a design (*without interpoles*), which has to be adapted for 500 volts. It should be noticed that it is the latter condition which, by limiting the turns per section, prevents a greater output from being obtained. This machine, however, should not be proceeded with until one or two ratios of $\frac{Y}{X}$ have been similarly worked through. We shall assume that this has been done, and that a general comparison of the results is in favour of the $11\frac{1}{2}$″ armature, so that we may pass to the calculation of the final details.

Armature Turns and Voltage.—With the number of commutator sections selected we have choice of the following numbers of armature conductors, using a two circuit winding :—

Turns per section.	Armature conductors.	Volts generated.	Speed.
1	222	100	1060
2	444	200	1060
3	666	240	848
4	888	400	1060
5	1110	480	1010

All of which are useful speeds and voltages; with lower speeds easily obtainable thus—

5	1110	240	500

Evidently the most serviceable armatures will be those having 3, 4, and 5 turns per section.

Arrangement of Slot.—Recalculating, we have—

$$\text{Equivalent pole-arc} = 6{\cdot}65''$$
$$\text{Teeth per pole} = 6{\cdot}8$$

Width of tooth root (with a maximum density = 154,000) = 0·338″

From these it is an easy step to the equation—

$$\text{Slot-depth} = 3{\cdot}75'' - 5{\cdot}9 \times \text{width of slot}$$

Whence the following table giving possible useful slot dimensions:—

TABLE XXII.

Slot-width.	Slot-depth.	Tooth-width at periphery.
0·375″	1·54″	0·6
0·4″	1·4″	0·575
0·4375″	1·16″	0·5375
0·45″	1·1″	0·515
0·46875″	1″	0·506
0·4875″	0·87″	0·4875

It will be noted that the figure given by the approximate formula (p. 184) is very near.

Arrangement of Conductors in Slot.—The most generally useful arrangement is that of Fig. 87; but it is evident that if round wire be adopted the same depth and width cannot be convenient for 3, 4, and 5 turns. This leads makers to select, and sometimes stock, more than one slot-shape for a given armature. If we keep as near as possible to our original design by adopting for this calculation 666 armature conductors, we must arrange the slot for three turns per section, 18 conductors per slot. Allowing the slot-linings given in Table IX., p. 144, with 7 mils of tape round each individual group of wires, and 7 mils of tape round each group of coils, we have—

Total thickness of insulation excluding wire-covering = 0·116″

Similarly—

Total depth of insulation exclusive of cotton covering	0·086
Separator between upper and lower coils	0·03
Binding wire space	0·060
Total	0·176

Now, the wires themselves evidently require a space proportioned three wide by six deep; *i.e.* 2 : 1.

The problem then is to choose that slot which minus 0·104″ in width

and 0·17″ in depth has a ratio depth to width of 2 to 1. In this way is the most economical shape of slot, in the case of wire-wound armatures, subservient to practical limitations. For these conditions the best slot is 0·48″ × 0·92″. For a 4-turn armature with appropriate insulation a slot $\frac{7}{16}$″ (= 0·4375) × 1·16″ would be better.

For a 5-turn armature also the latter proportions are better, so that possibly this slot might be chosen as the standard. If we assume that this is done, then the largest wire that can be got in with 250-volt insulation is No. 13 S.W.G., and that only with special fine covering. With the wider, shallower slot No. 12 would go in easily with special fine covering, or No. 13 very easily. No. 13 S.W.G. gives in the former case a space-factor of 0·24, and in the latter case 0·28.

Other points affecting the final choice of slot-dimensions are the standard slot-stamping tools available, as well as the number of machines requiring 5-turn armatures as compared with those requiring 3-turn armatures; and the possibility of adopting two small wires in parallel instead of one large one in the 5-turn case. The latter has the effect of considerably modifying the slot-dimensions, but it will not be found very convenient in this instance. Another adjustable coefficient is the ratio of nett to gross armature-length. This we have taken at 0·8, and a simple calculation shows that this will allow of a ventilating space 0·4″ wide. This latter might be reduced to 0·375″ or slightly less, and thus allow of a wider slot with the same tooth-density. It should also be remembered that where round wires only are likely to be used, as in the present example, the bottom corners of the slots can be slightly rounded, say with a radius of $\frac{1}{32}$″, and that then the slot can be deepened by the amount of this radius *without increasing the tooth-root density.* So that perhaps on the whole, if only one size of slot is to be standardized, a width of 0·45″ and a depth of $1\frac{1}{8}$″ with a $\frac{1}{32}$″ corner radius is the best solution. This allows of a wire for the 3-turn armature 0·096″ diameter with ordinary D.C.C., while the 4-turn armature will have a wire equivalent to No. 13 D.C.C., and the 5-turn armature may be wound with No. 14 or 15 D.C.C. according to the voltage.

Thus for the 3-turn armature we have—

Slot depth.—	Six wires (each 0·110 when covered)	0·660
	Various tapes	0·056
	Binding wire, etc.	0·062
	Slot lining	0·030
	Separator and making-up pieces . .	0·317
		1·125
Slot-width.—	Three wires	0·330
	Tapes	0·056
	Lining	0·060
	Allowance	0·004
		0·450

The slot is of course wasteful in depth, and the space-factor is only 0·27. It has been sacrificed to standardization.

In the five-turn armature we have—

Slot-height.—	Ten No. 14 D.C.C.	0·920
	Tapes	0·056
	Binding wire, etc.	0·062
	Slot-lining	0·050
	Separator	0·037
		1·125
Slot-width.—	Three No. 14/s.	0·276
	Tapes	0·056
	Lining	0·100
	Allowance	0·018
		0·450

Space factor = 0·29

The last figure is very good, but the slot is somewhat crowded for a high voltage. The "allowances" are for spring of the wire and variations in the insulation thickness.

These examples should suffice to illustrate the method of determining convenient slot proportions. It will be noticed that while theoretical ratios have little to do with the final determination, yet this latter is very near the preliminary figures under (13), (14), (15) above, as can easily be seen by placing the various results side by side.

General Conclusion.—The machine is now known to be satisfactory in all its important points, viz.—

Cost, ampere-turns of armature and of field, reactance-voltage, efficiency, constant losses, variable losses, temperature-rise, flexibility of output.

Final Calculations.—The following details should be finally calculated.

1. Ampere-turns per pole as carried out on pp. 41 and 56.
2. Length of armature winding as on p. 163.
3. Armature winding-pitch as on p. 114.
4. Commutator pitch as on p. 114.
5. Final iron losses from curves such as Fig. 16.
6. Temperature rise and output for the four- and five-turn armatures.
7. Commutator lengths for different voltages.
8. Actual back E.M.F. and speed for different armature-windings.

Range of Outputs.—The slot having been arranged to accommodate 3, 4, and 5 turns per section, it is desirable to tabulate the range of outputs obtainable. As on the five-turn armature the space-factor is good, there will be little object in varying the armature length, and the comparison suggested may be carried out by filling in a table like that given below, remembering that a two-circuit winding may be also connected as multiple-circuit.

TABLE XXIII.

DESIGN NO. . FLUX PER POLE IN THE ARMATURE, $1\cdot28 \times 10^6$ LINES; $D = 11\frac{1}{2}''$, $L = 4\frac{1}{2}''$; COMMUTATOR DIAMETER 7″; SECTIONS 111; ARMATURE SLOTS 37.

Turns per section.	Amps.	Volts.	Speed.	H.P.	Reactance volts, V_r.	Armature.		Field loss.	Commutator.	
						C^2R	Iron loss.		C^2R	Friction.
3	44·6	250	840	13	1·14	520	365	250	96	42
3	46·2	220	740	12	1·14	560	320	250		
4		250								
4		440								
4		500								
5		250								
5		440								
5		500								

From a series of tables like this it is possible to deduce the next most convenient size of machine which would then be proceeded with. In working out the list, however, one finds oneself repeatedly prevented from using the carcase for outputs which might be of use because of the reactance voltage limit, and thus one is led naturally to consider means for obviating this disadvantage. When making up the costs of the complete machine, also, one finds that the non-effective material, which in a machine of this size will cost about £7, will be about the same so long as the armature-diameter remains constant, whatever its length; so that one naturally inclines toward lengthening the armature to increase the output; but here again reactance voltage and field-copper cost intervene.

To a certain extent the latter is compensated for by the relatively reduced cost of non-effective material, and by the fact that a greater percentage of armature copper is rendered effective; so that up to the point where these conflicting conditions balance, the increased length of armature is undoubtedly an advantage.

This reasoning all tends toward the adoption of means for compensating armature-reaction and reactance-voltage, and the only cheap and effective possibility is the interpole.

Interpoles.—The justification for the use of interpoles in small machines not specially intended for wide speed variation is, then, that (*a*) being independent of reactance-voltage and of armature-reaction, the armature may be lengthened and the air-gap shortened until the reduced field-copper due to the latter, and the relatively reduced cost of end plates, etc., show a substantial advantage over the increased L.M.T. of the field, the cost of the interpoles, and the increased armature material; so that at the increased output obtained the cost per K.W. is substantially reduced.

Application to 13-H.P. Motor.—In the present instance the procedure might be somewhat as follows:—

Few makers would care to reduce the air-gap, though in the author's opinion this might safely be made $\frac{1}{16}''$, thus saving 600 ampere-turns per shunt field-coil.

The present cost of net effective material is £0·71 per H.P., or adding £7 for non-effective material, we get £1·25 per H.P.

The amount by which the armature can be economically lengthened is largely a matter of trial and error, and we may commence by trying an increased gross length of 50 per cent. The poles must now be made elliptical or rectangular in section. Adopting the latter, we get—

$$L = 4·5 \times 1·5 = 6·75$$

Flux per pole in the armature $12·8 \times 10^5 \times 1·5 = 19·2 \times 10^5$

If one air-duct through the armature be still used, the new net length becomes—

$$(6·75 - 0·4) \times 0·9 = 5·7''$$

$$\text{Ratio net length : gross length} = 0·84$$

So that the densities in teeth and armature are reduced almost in the proportion 0·8 : 0·84. Whence, as a first approximation, the ampere-turns for gap and teeth become 2000, and the total shunt ampere-turns per pole are 2400 (cf. p. 187).

*(2) Pole-area = $16 \times 1·5 = 24$ sq. in.
(6) Pole axial length = $6·75''$
Pole-thickness = $3·55''$

(32) Allowing the same shunt current as before (viz. 1 ampere), we find that about 2400 turns per coil of the same-sized wire (No. 20 S.W.G.) will, in a coil $3\frac{1}{4}''$ long $\times$ $1\frac{3}{4}''$ deep dissipate the heat readily, and require about 30 lbs. of copper per coil. The rectangular coil acounts for this increase in copper for the same loss, the L.M.T. being nearly 30″. The same pole-length will therefore be used, and the diameter of yoke will remain the same, its section, and therefore its weight, being increased by 50 per cent. to maintain the density constant.

Armature Output.—At 840 r.p.m., with three turns per section and $19·2 \times 10^5$ lines per pole, the generated back E.M.F. will be 358 volts. L.M.T. of armature becomes $38\frac{1}{2}''$.

(17) $R_a = 0·31$ ohm.

(31) Temperature rise calculations show that the armature will dissipate about 1200 watts.

(23) Weight of stampings neglecting slots = 115 lbs. approx.
(24) Frequency = 28
(25) Iron loss = 486 watts
(18) Copper loss in armature = (1200 − 486) = 714 watts
Maximum armature current = at least 50 amps.
Output = 358×50 = about 24 H.P.

* The reference numbers apply to the previous calculation.

Output of the Five-turn Armature.—A simple calculation shows that at a speed of about 800 r.p.m. the machine will give, on 500 volts with five turns per section, about 18 H.P. This corresponds to about 2000 armature ampere-turns per pole and a reactance-voltage $V_r = 2{\cdot}5$. Both these show the necessity for interpoles. Evidently to neutralize armature reaction alone, each interpole must carry 2000 ampere-turns. Besides this we must set up the commutation flux.

Recalculating the five-turn armature of p. 192, we obtain: length of mean armature turn = $38\frac{1}{2}''$.

(17) Armature resistance, 0·84 ohm; resistance of one armature coil, 0·03 ohm.

Calculating the value of f as on p. 123, we get $f = 1020$ if the maximum number of turns short circuited = 3. This corresponds to a brush $\frac{1}{2}''$ wide.

From this we obtain $L_c = 0{\cdot}000255$
also with a brush $\frac{1}{2}''$ wide $t_c = \frac{3}{1500}$ seconds

$$\text{whence } \frac{r}{L_c} t_c = 0{\cdot}23$$

From Fig. 80 $\epsilon^{-\frac{r}{L_c}t_c} = 0{\cdot}8$

Since $C_w = 15{\cdot}5$, we have—

$$e = 4 \text{ volts}$$

Substituting this in the formula on p. 123, we get—

$$\text{Interpole flux} = 8 \times 10^4 \text{ lines per pole}$$

As a first approximation take the leakage factor for the interpole itself as 1·8, and allow a density of say 100,000 lines in the pole, then the diameter of the interpole is say $1\frac{5}{16}$.

The ratio of the interpole-arc to the armature circumference should be at least as great as that of the brush width to the commutator-circumference, so that the coil may be under the interpole shoe during the whole time of commutation.

In this case the pole-arc may be 1″, so that the pole will slightly overhang the shoe, but this is of no consequence.

Taking a density under the shoe of 40,000 lines per square inch gives us an axial length of shoe of about 2 inches. The pole, therefore, is formed much as in Fig. 123, but the neighbouring pole-shoe is not cut away. The interpole air-gap may be the same as the gap under the main poles, *i.e.* $\frac{3}{32}''$, and the ampere-turns may be calculated exactly as for the main poles.

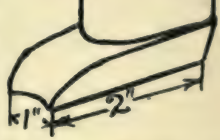


Fig. 123.—Sketch of Interpole and Shoe.

We thus find the ampere-turns for the air-gap = 1174, and allowing 12 per cent. extra for the iron path gives 1408 ampere-turns for the interpole, or say 1400.

The armature cross-ampere-turns are 2000, so that the total ampere turns per interpole = 3400.

$$\text{Full-load current} = 31 \text{ amps}$$
$$\text{Turns per coil} = \frac{3500}{31} = 110$$

Interpole Loss.—A rough calculation of the efficiency shows that in this case we can afford to take the loss at 1 per cent. or 140 watts.

As the interpole-coils will not be taped, and because they are not deep, C_h may be taken at about 150.

Carrying out the calculation for temperature rise exactly as for shunt field coils, we find that 110 turns per pole of No. 7 S.W.G. D.C.C. will give a loss of about 130 watts and a temperature rise of about 55° Centigrade mean.

Losses and Efficiency.—Checking out now the losses and efficiency, we get—

Armature input = 31 × 500 =	15,500
Field loss =	250
Total input . .	15,750 watts

Armature copper loss =	814
Iron loss =	386
Shunt field =	250
Com. losses and friction =	320
Interpole loss =	140
Total losses . .	1,910
Output . .	13,840 = 18½ H.P.

Efficiency = 88%, which agrees well with Fig. 19.

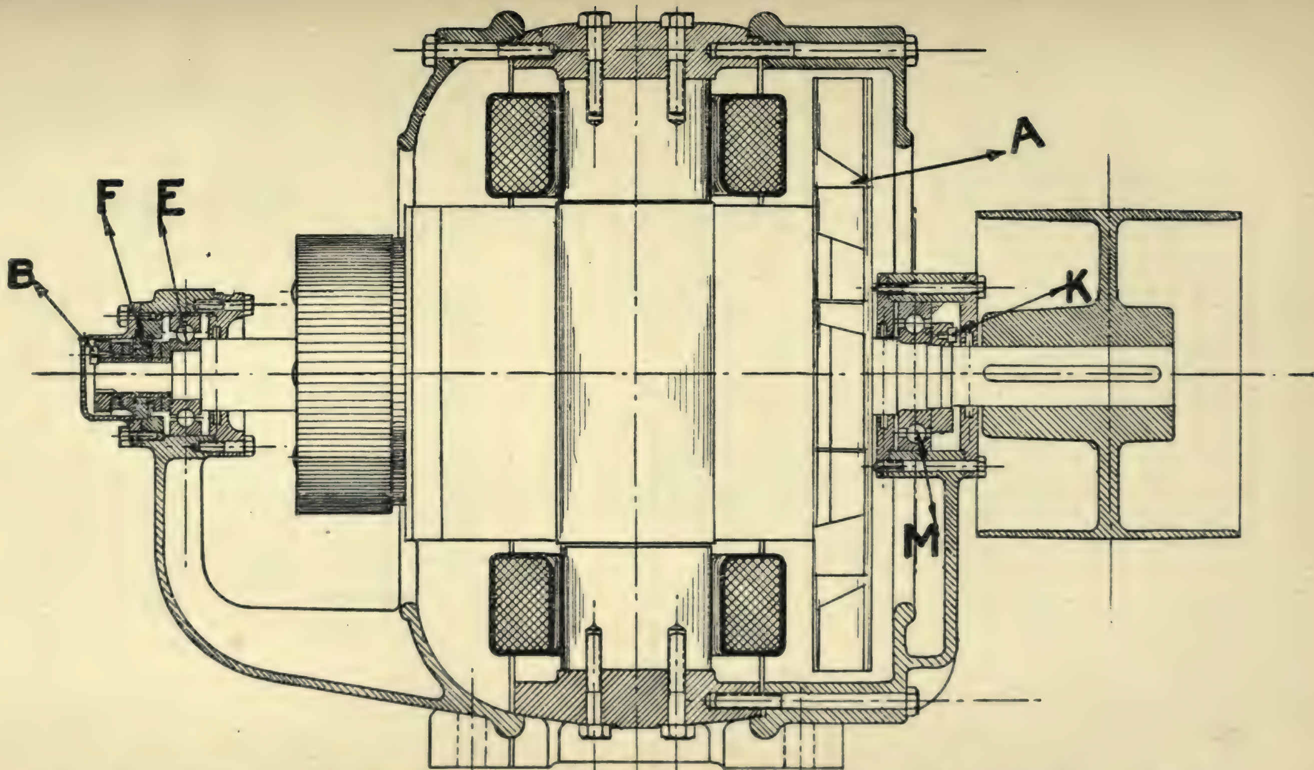
Possibility of enclosing.—

Total variable losses including interpoles = 1050
Total constant losses = 860
Ratio constant : variable losses = 0·815

This is a great improvement on the non-interpole machine, where the ratio was 1·7 (see p. 186). The result of this low ratio is that the machine will have its maximum efficiency at a load less than full load, and it will be almost as good enclosed as when open.

Effect on costs.—The costs for this machine come out about as follows:—

	£
Armature iron	1·9
Armature copper	1·36
Steel poles	0·64
Iron yoke	2·62
Shunt field-coils	3·93
Commutator (not shortened for 500 volts) .	1·25
Interpole coils	1·3
Total cost of effective material . .	13·00



B. Locking plate. F. Thrust bearing. E. Main bearing. M. Main bearing. K. Locking plate. A. Fan.

FIG. 124.—MODERN MOTOR WITH BALL BEARINGS AND FORCED VENTILATION.

If the cost of non-effective material be again taken at £7, we have a total material cost of £20, or, at a speed of 840, £0·83 per H.P. as against £1·26 per H.P., which was the best obtainable without interpoles.

If we take the ratio of total works cost to cost of net effective material as 3·25, the total works cost becomes £42, and the selling price to-day (1910) is about £52. So that the addition of the interpoles has reduced the specific cost without sacrificing the efficiency.

Final calculations.—It is to be noted that although this last machine may be taken as a fair example of modern practice, the designer should not rest satisfied with it until he has assured himself that it is the best that can be done. Further, it is seen that the best shape of machine without interpoles is different from the best shape with interpoles, so that to take a standard machine and add interpoles is a mistake.

The use of ball bearings in small machines is coming into favour, as it raises the efficiency considerably, because it halves the friction loss and thus saves a certain amount of heat which would otherwise have to be dissipated.

Of course, before any machine is put in hand all the figures should be most carefully rechecked, especially as regards temperature rise, upon which an interpole design entirely depends. The author thinks that with carefully arranged ventilation or by the use of a fan mounted on the armature shaft (Fig. 124), even the figures above obtained could be still improved by 5 or 10 per cent. Possibly a slightly lower tooth density and longer air-gap would be preferred by many makers.

Constant Pressure Machines.

Problem II.

As a further example of constant-speed machines we will consider the main dimensions of *a 200-K.W. generator with interpoles to run at* 400 *r.p.m.* This example will be the more interesting since we have already made use of a machine of this output *without interpoles* to exemplify formulæ and statements in various chapters.

We have thus obtained—

From the commutation limit (p. 132), $D = 35''$ for circular poles

With 4 poles $L = 15''$	$D^2L = 18{,}500$
With 6 poles $L = 10''$	$D^2L = 12{,}200$
With 8 poles $L = 7\frac{1}{2}''$	$D^2L = 9100$

The latter figure agrees with those obtained from temperature considerations on pp. 28 and 85, assuming that with 8 poles the division of losses and efficiency was still about the same as for the 4-pole case; so that the number of poles in non-interpole machines is often determined by the problem of getting the dimensions demanded by commutation to coincide with those required by temperature-rise.

Type of Armature Winding.—The question of the type of

armature-winding has some effect upon the number of poles. In non-interpole machines of this size two-circuit windings are usually very unsatisfactory, and for 500 volts a multiple-circuit winding is out of the question with 8 poles, on account of the large number of conductors required. Interpole machines, on the other hand, can be arranged with a two-circuit winding for almost any output. This also affects the space factor. For the fewer bars per slot required by a two-circuit armature render a higher space factor possible (see p. 146).

Deductions from Standard Makers.—It is evident that the question of the number of poles to be used is one of the difficult points to decide in such a case as this. If, as in Problem I., we compare some similar-sized machines of different makers this is further emphasized. For instance, take the two following interpole machines by first-class makers :—

	A	B
Output K.W.	200	200
Volts	500	500
Speed r.p.m.	400	400
Poles	8	4
D (inches)	$34\frac{1}{2}$	$27\frac{1}{2}$
L (inches)	$10\frac{1}{2}$	13
D^2L	12,500	9850
Peripheral speed	3600	2900
ratio $\frac{\text{pole-arc}}{\text{pole-pitch}}$	0·75	0·7
$\frac{Y}{X}$	400	750

Here it is apparent that for machines of similar output the two makers have decided upon very different designs. From the point of view of *active* material alone B would probably be dearer than A; but the smaller amount of inactive material in B (due to the smaller diameter) might possibly compensate for this. It will be noticed that the value of $\frac{Y}{X}$ in the former case agrees with the limits mentioned on p. 62, but the latter is far higher.

Assumed Densities, etc.—Since this design is to be fitted with interpoles, we may choose the tooth-densities and gap-length rather lower than otherwise would be permissible. We take—

Material of yoke and poles	cast steel
Poles	circular
Shoes	laminated
Air-gap (Fig. 13)	0·125″
Voltage	500
Winding	two-circuit

Slot space-factor	0·38
Temperature rise on full load .	40° C.
Efficiency	93% to be a maximum at $\frac{3}{4}$ full load
Density at the pole-shoe face .	50,000
Maximum tooth-density . .	140,000
Density below teeth . .	75,000
λ	1·15 (assumed)
Yoke-density	80,000
Pole-density	100,000
$\frac{\text{Pole-arc}}{\text{pole-pitch}}$	0·7

Total generated E.M.F.—The total copper-loss in armature and interpoles will probably not exceed 3 per cent. of the output. If, then, we reckon on a maximum flux corresponding to 520 volts, we ought to be on the safe side.

Relationship of D and L.—From the densities above given and in accordance with the analysis on p. 20, we obtain—

$$D = \frac{pd}{1{\cdot}61}$$

Depth of Slot.—Substituting in the formula on p. 15, we get—

$$\text{Depth of slot} = 0{\cdot}05D$$

Important Ratios.—These are the same as in the previous example (p. 183), except that V_r is of little importance since the machine has interpoles.

Value of $\frac{Y}{X}$.—We have—

$$Y = \frac{\pi d^2}{4} p \times 10^5$$

and $$X = w\frac{C}{2}$$

But $$w = \frac{8000E}{\pi d^2 np} \text{ (neglecting } \lambda)$$

whence $$\frac{Y}{X} = \frac{d^4p^2}{480} \text{ approximately}$$

Selecting now a trial value for $\frac{Y}{X}$ in accordance with p. 62

$$\text{Say } \frac{Y}{X} = 400$$

$$\text{We get } d^2p = 440$$

whence if $L = d$, we obtain the following table for likely numbers of poles :—

TABLE XXIV.

Poles.	D.	L.	D^2L.	Peripheral speed.
4	26″	10·5″	7,100	2720
6	32″	8·6″	8,800	3300
8	37″	7·5″	10,200	3840

It is evident that the problem of the right number of poles to adopt is the first that arises in a machine of this size.

The choice of the above possible machines must be decided from a consideration of temperature-rise and cost. We therefore proceed to analyze the division of losses for the three cases.

Losses.—Since the efficiency is to be 93 per cent. at $\frac{3}{4}$ load, we obtain from p. 85—

Constant losses	5250 watts
Variable losses	9300 ,,

We further assume, as on p. 30—

Friction of bearings and windage . 1200 watts

To obtain an approximation to the iron losses, we tabulate as follows :—

TABLE XXV.

Design No.	I.	II.	III.
Poles	4	6	8
D	26″	32″	37″
d	10·5″	8·6″	7·5
Flux in pole	$86·5 \times 10^5$	58×10^5	44×10^5
Slot-depth	1·3″	1·6″	1·85″
Flux in armature . . .	75×10^5	50×10^5	$38·2 \times 10^5$
Section below teeth . . .	50	33	25·5
Net length	8·4″	7″	6″
Radial iron depth including slot	7·3″	6·5″	6·1″
Volume of core neglecting slots .	3630	3680	4040
Weight, lbs.	1010	1050	1130
Frequency	13·3	20	26·6
Constant (p. 29)	1·7	1·7	1·7
Iron loss	1720	2700	3850

Variable Losses.—With interpoles a comparatively soft brush may be used, as, say, Battersea B type with a voltage drop at the commutator of 1. Thus the commutator copper loss is $2 \times 400 = 800$ watts, and the interpole loss may be taken provisionally at 0·3 per cent., *i.e.* 600 watts (see p. 124). Subtracting these items from 9300 gives an armature copper loss of 7900 watts. Adding this to the iron losses just calculated, we obtain—

TABLE XXVI.

Design number.	I.	II.	III.
Total watts armature must dissipate	9,620	10,600	11,750
Method 2, p. 82, A = . . .	56	52	50
Method 2, p. 82, T = . . .	56°	46°	42°
Watts armature will dissipate, Method 3, p. 82	7,200	9,500	10,400
Length of mean armature-turn .	79″	63″	55″
Armature conductors . . .	520	520	510
Aggregate slot area . . .	53	80	107
Net copper area	20·5	30·4	40·6
Area per conductor . . .	0·042	0·064	0·086
Armature resistance loss at 55° .	15,100	7,800	5,100

Comparing the last figures with the copper loss calculated from the efficiency (viz. 7900 watts), and noting the inferences to be drawn from the temperature rise figures, we see that the four-pole design is out of the question. Either the six- or eight-pole machine will, with slight modifications, evidently yield the output, and both should be worked through. On account of its much smaller diameter and iron loss, we select here the former as being the more likely, and proceed to adjust the discrepancies by a slight increase of armature-length, pole-diameter, and slot-area.

It is to be noted that in rough preliminary calculations of machines as large as this the approximate iron loss formula of p. 29 may still be used as it was throughout in the case of the smaller machines. For subsequent more accurate approximations, we shall make use of the formula on p. 30, in which the iron loss in the teeth can be separated from that in the armature body.

TABLE XXVII.

Second approximation.

Six-poles. 200 K.W. 400 r.p.m. 500 volts.

(8)	D	32″
(1)	d	9″
(2)	Area pole-core . . .	63 sq. ins.
(3)	Flux per pole . . .	63×10^5
(4)	Flux issuing from shoe . .	55×10^5
(5)	Area of shoe face . . .	110 sq. ins.
(6)	Axial shoe length . . .	9″
(7)	Pole-arc	12·3″
	Ratio pole-arc to pole-pitch .	0·73
(9)	$\frac{pd}{D}$	1·69
(10)	Armature conductors . .	474 (nearest even number)
(11)	Ampere wires per inch . .	950
(12)	D^2L	9220
(13)	Depth of slot . . .	1·6″
(14)	Net copper area . . .	30·4 sq. ins.
(15)	Area per conductor . .	0·064
(16)	Length of mean turn . .	64·5″
(17)	Armature resistance (hot) .	0·0465 ohm
(18)	Armature resistance loss .	7450 watts
(19)	Voltage drop due to resistance	18·5
(20)	Section of armature below teeth	37 sq. ins.
(21)	Radial armature depth below teeth	5″
(22)	Internal diameter of stamping	18·8″
(23)	Weight of armature core excluding teeth and slots .	760 lbs. (2700 cub. ins.)
	Weight of teeth . . .	150 „ (530 cub. ins.)
(24)	Frequency	20
	Hysteresis loss in armature body	1010 (coefficient = 0·003)
	Eddy loss in armature body .	340
	Hysteresis loss in teeth .	550
	Eddy loss in teeth . .	20
	Calculated iron loss . .	1920

The latter is in this instance a good deal below that given by the approximate formula. In the author's opinion 1920 is too low, and 2800 too high.

The whole subject deserves much closer consideration than it has hitherto received.*

As a margin, then, for possible error, we shall allow an increase of 10 per cent. over the last calculated value, and estimate the final iron loss at 2200 watts. Thus—

(25)	Estimated iron loss	2200
(26)	Turns per armature section . . .	1

(27) **Number of Teeth.**—This, as well as the actual slot dimensions (No. 13), depends upon conditions rather different from those referred to on p. 190. For in a machine of this size round wires would seldom be used, and consequently the sectional shape of the armature conductor can be made to suit the best shape of slot instead of *vice versâ*. We know that the number of conductors will be about 474, that the number of slots will be even and of the order of 4D (= 128), that the fewer the slots the better, and that the number of conductors must suit the formula—

$$W = py \pm 2$$

Now, if $y = 79$, $W = 476$ or 472, which is the nearest approach to 474. We then obtain—

y	W	Slots possible.	Conductors per slot.
79	472	118 or 59	4 or 8
	476	119	4

If we select 59 slots, there will only be about 7 teeth under each pole-arc, and the ampere-wires per slot would be about 1600. Such an arrangement (unless the slots were half closed) would lead to great field oscillation as each tooth passed under the pole; also eddy-currents, humming, and possibility of sparking would result. We thus decide upon 118 or 119 slots. The actual choice would of course, in practice, depend upon the division plates available on the stamping machines. We decide on 118 slots.

(28)	Commutator	sections . . .	236
	„	diameter . . .	20″
	„	peripheral speed . .	2100 f.p.m.
(29)	„	friction loss . .	570 watts
(30)	„	C^2R	800 „

* Very serious discrepancies are sometimes found between computation and test, which have never been properly examined. Thus the approximate formula given on p. 29 shows that the eddy loss is almost negligible compared with the hysteresis loss, which is also borne out by the values just calculated. On the other hand, some writers declare the eddy loss to be far greater than that due to hysteresis. Thus the author's formula on p. 30 for eddy currents gives results almost four times those of S. P. Thompson's formula (*Dynamo Electric Machinery*, vol. i., 1904 ed., p. 104). And the values given by Hawkins and Wallis (*The Dynamo*, 1903 ed., p. 633) are for eddy currents far greater than even the author's formula would yield.

Choice of Slot Dimensions.—The formula upon which the provisional slot-depth has been based is dependent upon the ratio $\frac{\text{tooth-width}}{\text{slot-width}}$, as shown on p. 15. If this be varied, a different depth and area of slot results even when the tooth density is maintained constant. In non-interpole machines a great depth of slot leads to excessive self-induction with consequent sparking. In interpole designs, however, this limit does not exist. If for m_1 on p. 15 we substitute different ratios we shall get different slot depths, and now that the number of teeth is decided the corresponding width will be known, so that we can tabulate as follows :—

Slot-pitch = 0·85″ at armature circumference

$m_1 = 1$	slot-depth = ·05D = 1·6″	slot-width = ·425″	slot area = ·68 sq. in.
„ = ·8	„ = ·078D = 2·5″	„ = ·378″	„ = ·94 „
„ = ·6	„ = ·112D = 3·6″	„ = ·32″	„ = 1·15 „

It is then seen that taking a value of m_1 less than unity leads to a larger slot area; and since the space-factor is constant and the shape of conductor is (within limits) adjustable, this is a very clear advantage. On the other hand, too great a slot depth may lead to so much flux-leakage across the slot, particularly if the tooth-density be high, that the flux embraced by the lower conductors will be sensibly less than that embraced by those at the top of the slot, causing a diminution of E.M.F. and the possibility of eddy-currents in the armature-bars.

Maximum Slot-area.—From the formula on p. 15 it is easy to find the maximum possible slot-area now that D and the densities and the tooth-pitch are known. For since K = 0·405, we have—

$$\text{Depth of slot} = \frac{1 - 0{\cdot}405(1 + m_1)}{2(1 + m_1)} \times 32$$

$$\text{Width of slot} = \frac{0{\cdot}85 m_1}{1 + m_1}$$

The product of these two is the area of the slot, which may be written—

$$\frac{13{\cdot}6 m_1}{(1 + m_1)^2} - \frac{5{\cdot}55 m_1}{1 + m_1}$$

The differential coefficient of this expression with respect to m_1 is—

$$\frac{8{\cdot}05 - 19{\cdot}15 m_1}{(1 + m_1)^3}$$

Since the differential coefficient of the latter with respect to m_1 is negative, when equated to zero it will give the value of m_1 corresponding to the maximum area of slot. We thus obtain—

$m_1 = 0{\cdot}42$, slot-depth $= 0{\cdot}15\text{D} = 4{\cdot}8''$, slot-width $= 0{\cdot}25''$, slot-area $= 1{\cdot}2''$

So that up to a ratio $\frac{\text{depth}}{\text{width}} = 19{\cdot}2$, the area of the slot constantly increases.

Now, such a depth as 4·8″ has never been tried. For besides the

disadvantages mentioned on the previous page, the bore of the armature would be so much reduced as to interfere seriously with the ventilation. Certainly with interpole machines designers should move in the direction of deeper slots, but such progress must be made with extreme caution. Thus in the present instance the author would consider any value of m_1 less than 0·9 as of an experimental nature. The size of slot we decide upon is then $1\frac{5}{8}'' \times \frac{7}{16}''$, which corresponds to a value of $m_1 = 0·98$ about. There is no object in this case in rounding the bottom corners of the slot much, because with rectangular bars very little is gained in space factor thereby.

Slot Insulation.—The lining of the slot may be composite and arranged as generally indicated in Table X., p. 146.

We thus have a thickness on either side of the slot of say 50 mils. If the bars are covered with a combination of double cotton and braiding, a thickness of 0·015″ on either side must be allowed. Since there are four bars per slot, they may be arranged as in the sketch, Fig. 89, 2, p. 145, from which it is clear that the maximum thickness of copper per bar will be 0·138″. It is always necessary to allow something for lack of uniformity in the bars, so that taking 0·014″ for this we obtain 0·12″ as the maximum thickness of each bar. In height for the wooden wedge and clearance 0·22 inch will be ample. Beneath this a strip of micanite 0·015″ thick may be placed, while between the upper and lower bars a composite strip 0·03″ may be inserted. This leaves, when irregularities in the bars are allowed for, a depth of about 0·6″ per bar.

A net area of copper per bar of 0·072 sq. in. is the result of these adjustments, comparing very favourably with the value 0·064″ in Table XXVII

Space Factor.—The space factor becomes as nearly as possible 0·4 instead of the value 0·38 allowed. This increase is largely due to the use of a two-circuit winding, which requires only 4 bars per slot. With the much larger number required by a multiple-circuit winding the space factor would have been less than 0·38.

Losses.—The corrected values are as follows :—

Variable.—	Armature copper-loss	6550	
	Commutator ,,	800	
	Interpole	600	
		——	7950
Constant.—	Armature iron loss	2200	
	Bearing friction	1200	
	Commutator friction	570	
		——	3970
	Total losses excluding shunt-winding		11,920
	Constant losses allowed	5250	
	Shunt loss allowable	1280	

Armature Heating.—Total power to be dissipated as heat 8750

From Table XXVI. it is evident that by any method the temperature

rise will be on the safe side, and if the efficiency is to be a maximum at $\frac{3}{4}$ full load, one may increase the output by an amount corresponding to a variable loss of 1300 watts. This is an increase of nearly 8 per cent., *i.e.* the output becomes 215 K.W.

Field-dimensions.—The pole-area and diameter being settled, and also the section of the yoke and the dimensions of the pole-shoe, there remains the length of the field-pole. This is determined, as in previous examples, by approximating to the field ampere-turns and deducing the area required from temperature rise.

Gap Ampere-turns.—		
Pole-pitch	16·7″	
Pole-arc	12·3″	
Distance between pole-tips	4·4″	
Length of air-gap	0·125″	
Constant (Table III. p. 40)	3·9	
Effective pole-arc	12·79	
Effective pole-arc area	115 sq. in.	
Density in gap (no slots)	48,000	
$\dfrac{\text{Tooth-width}}{\text{slot-width}}\left(\dfrac{1}{m_1}\right)$	1·02	
$\dfrac{\text{Slot-width}}{\text{gap-length}}$	3·5	
Constant from Fig. 26, p. 41	1·27	
Actual air-gap density	61,000	
Gap ampere-turns		2400
Teeth Ampere-turns.—		
Teeth per effective pole-arc	15·2	
Tooth-width at top	0·4125	
Tooth-width at root	0·3275	
Mean density	136,000	
Ampere-turns (see Fig. 27)		2600
Ampere-turns for gap and teeth		5000
Armature-core Ampere-turns.—		
Density in core	75,000	
Length of mean line	12″	
Ampere-turns (Fig. 6)		120

If as a first approximation we allow 20% of the above total for the yoke and pole, we must provide altogether for, say, 6200 ampere-turns per pole.

Allowing 10% for the P.D. across the shunt rheostat gives $\frac{450}{6}$ or 75 volts across each field coil; this corresponds to a shunt current of $\frac{1280}{500}$, or, approximately, 2·56 amps. Shunt turns per pole $= \frac{6200}{2\cdot56} = 2420$. Thus, if the calculated ampere-turns be correct the loss in the field coils will be 1140 watts approximately, and in the rheostat 150 watts. Thus watts per field coil $= 190$.

Field-coil Dimensions.—The number of ampere-turns being small (because of the interpoles), it will probably be possible to avoid using a

ventilated coil. We try, therefore, first, Case II., p. 75, and assuming the coil to be wound on a metal former without taping, we adopt—

$$C_h = 140 \text{ (p. 72)}, \ T = 60^\circ$$

$$\text{Thus } A_m = \frac{C_h P_m}{T} = \frac{140 \times 190}{60} = 450 \text{ sq. ins.}$$

Since $d = 9''$, $r_2 = 5''$, allowing for the spool, and the two equations for the coil are—

$$450 = 4{\cdot}7d_c^2 + 31{\cdot}4l_c + 6{\cdot}28l_cd_c + 62{\cdot}8d_c \quad . \quad . \quad (1)$$

$$l_cd_c = \frac{4}{\pi}\cdot\frac{82\,.\,6200\,.\,6200\,.\,\pi\,.\,(10 + d_c)m^2}{10^8 \times 190} \quad . \quad . \quad (2)$$

If m^2 be taken provisionally at 1·6, (2) becomes—

$$l_cd_c = 1{\cdot}1(10 + d_c)$$

The values $l_c = 7''$, $d_c = 1\frac{3}{4}''$ satisfy these equations nearly enough.

We thus obtain a pole length of 8″, and the field-winding consists of 2500 turns per pole of a wire about 0·054″ diameter. This is between Nos. 18 and 17 S.W.G., though so near the latter that probably No. 17 would be adopted with rather more shunt resistance.

Yoke Dimensions.—Allowing 1″ for the thickness of the pole shoe in the centre, and adding the length of pole and air-gap, we get as the minimum internal diameter of the yoke $50\frac{1}{4}''$. It will be necessary, however, as the field coils are circular, to leave a flat facing, on the inside of the yoke above the pole, and the proper thickness of this is best obtained by setting the machine out on a drawing-board. We shall not, however, be far wrong in taking the actual internal diameter of the circular part of the yoke at 4′ 4″. The yoke density being 80,000 gives the yoke section as about 40 sq. in. Taking a width of 14″ gives us a mean thickness of about $2\frac{7}{8}''$. The main over-all dimensions are now complete with the exception of the interpoles, so that the total ampere-turns and the value of the leakage factor can be checked.

Interpole Dimensions.—We shall assume in this example that the patents mentioned on p. 151 are not to be used, and that therefore the axial length of the interpole shoe must be the same as that of the main shoe, *i.e.* 9″. We have—

Thickness per commutator section . .	0·265″
Brush width	0·5″
Sections short circuited	2
Time of commutation	0·00127 sec.
Turns per segment	1
f (see p. 122)	460
L_c	0·0000092
r	0·0007
$\frac{rt_c}{L_c}$	0·097
$\epsilon^{-\frac{r}{L_c}t_c}$ (Fig. 80)	0·93

e (p. 121)	3·86, say 3·9 volts
Interpole arc	$\frac{7}{8}''$
Time coil is under pole, allowing fringing factor = 1·15	0·0015 sec.
Corresponding flux necessary	292,000 lines
Interpole shoe area with fringe . . .	9 sq. ins.
Approximate average air-gap density, allowing for increase due to teeth (p. 41)	$1{\cdot}27 \times 32{,}000 = 41{,}000$
Ampere-turns for gap	1600

Interpole Ampere-turns.—The width of the interpole arc and the flux required are here obtained in exactly the same way as for the smaller machine. On account, however, of the greater axial length of interpole-shoe, the leakage factor for the interpole will be very high. The ampere-turns per pole on the armature to be neutralized are nearly 7900, so that the total interpole ampere-turns cannot be much less than 9000. These act in conjunction on one side with the ampere-turns of the main pole, which are estimated to be about 6200, so that at least 15,000 ampere-turns are acting across the gap between the two shoes.

Now—

Interpole-arc = $\frac{7}{8}''$

Distance between main pole-shoes = 4·3″

Distance between interpole and main shoe = 1·7 nearly

Density across 1·7″ due to 15,000 ampere-turns = 30,000

If we assume that the area across which this leakage takes place is 9″ long by $\frac{3}{4}''$ wide, we get—

Total leakage flux = 200,000 lines at least

Now—

Commutating flux = 292,000

So that the leakage due to the shoe alone corresponds to a leakage factor of at least 1·66, and therefore we must assume the factor for the whole of the leakage flux to be not less than 2.

Thus—

Flux in the interpole = 600,000

Interpole density = 90,000

Interpole area = 6·7 sq. ins.

Interpole diameter = 2·92 ins., say 3 ins.

Now, on account of the rather small space between the poles at the inner end, and assuming that the interpoles are to be forgings, it is more convenient not to adopt the circular section. Instead we may take a section 2″ thick and $3\frac{3}{4}''$ long, with the ends of the section rounded so that they are almost semicircular. The ampere-turns for the interpole are then as follows :—

Reversing armature ampere-turns	7866
Ampere-turns for gap-flux	1600
„ „ iron-circuit	160
	9626

Turns per interpole $\frac{9626}{400} = 24\frac{1}{2}$

To obtain this number of ampere-turns with a loss of 100 watts per coil entails a resistance per coil of $\frac{1}{1600}$ of an ohm. It will be necessary, therefore, to adopt a strip of about 0·37 sq. in. section, and this winding will, even with the interpole core shaped as above, barely clear the main field coil windings at the inner end. It may be necessary, therefore, either to wind the interpole coil taper so that it is deeper at the yoke end where there is plenty of room, or to alter the shape of the main poles slightly so as to give rather more clearance.

Both methods are in use, and many examples of either are to be seen among modern machines. These, however, are all details which would not be definitely decided upon until the final design for the output was chosen. It is sufficient to see that the coils can be made to clear (as is the case in this instance), when the table of approximate costs can be proceeded with as on p. 188 and p. 196.

Approximate Cost of Net Effective Material.—Using the prices given on p. 175 as a guide and working out the various parts of this preliminary design we get the following list of costs :—

	£	s.	d.			
Iron—						
Yoke	20	0	0			
Poles	9	0	0			
Shoes	2	0	0			
Interpoles	6	0	0			
Armature core	19	0	0			
				£56	0	0
Copper—						
Armature	14	0	0			
Shunt-field	22	0	0			
Interpole copper	8	15	0			
Commutator	20	0	0			
				£64	15	0
Total cost of effective material				£120	15	0

Assuming the ratio of total works cost to a cost of effective material as 2·5, the total works cost of this machine would be £300. The present market price (1910) is about £340.

Final Design.—Having completed a preliminary design as above, other sets of figures on similar lines can be worked out starting with slightly different ratios. From these the most economical machine can be selected, and the final details of the field completed very much as has already been done for the small machine in Example I.

Series-wound Machines.

Generators for constant E.M.F. are shunt or compound wound. Series winding is used for (1) generators, where variable E.M.F. and constant

current are required; (2) motors, where variable speed and torque are required.

Instances occur where a variable E.M.F. is required with a constant current. This is so in the case of arc lamps in series for street-lighting. To meet these demands, machines have been invented which will automatically adjust themselves to the correct P.D. at constant current.

Principle on which Constant-current Machines work.—The characteristic of a series-wound generator running at constant speed is as shown (Fig. 125). The machine of course only excites when the circuit is closed.

A shunt generator gives a more constant E.M.F., as shown in Fig.

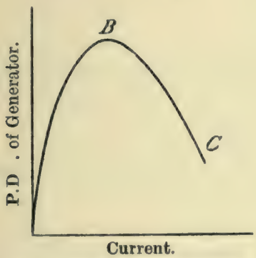


FIG. 125.—SERIES CHARACTERISTIC.

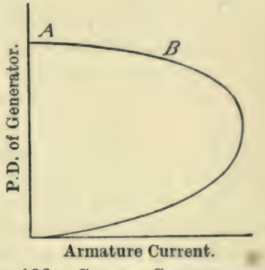


FIG. 126.—SHUNT CHARACTERISTIC.

126, the only part of the curve used in practice being from A to B. Beyond B the voltage is very unstable.

Below is shown a circuit with arc lamps connected in series. With such a connection a lamp must be short-circuited to be cut out. The current then tends to rise. But for arc lamps an almost constant current is required, so that the P.D. must be decreased to check this rise. For

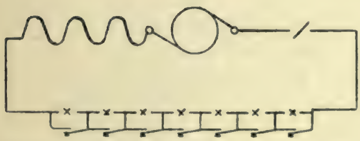


FIG. 127.—SERIES-WOUND GENERATOR WITH ARC LAMPS IN SERIES.

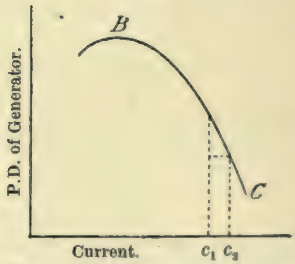


FIG. 128.—CONSTANT-CURRENT CHARACTERISTIC.

these conditions, then, the part BC of the curve in Fig. 125 would be suitable, since the P.D. decreases as the current rises. With a view to using this part of the curve, machines have been designed to give a rapid fall from B to C, this drop occurring at the current required. Then for a small change in current from c_1 to c_2, Fig. 128, we get quite a large

change in P.D. If by short-circuiting a lamp the current tends to rise, a very small increase of current quickly brings down the P.D. to the value required. This machine is thus to a certain extent self-regulating, the change in current only being small, and full armature current is obtained on complete short circuit only, *i.e.* at "no" P.D.

Design of Constant-current Machines.—We require a machine which shall possess as steep a characteristic as possible between B and C, Fig. 128. Keeping in mind the ordinary series generator, we remember that this fall BC is due to—

(1) Saturation, which brings the curve horizontal, and
(2) Armature-reaction, which in destroying the main field brings the curve down.

Hence, in designing, it is necessary—

(1) To have at the maximum P.D. all the iron parts saturated.
(2) To arrange the machine so that from B to C the armature-reaction is heavy, *i.e.* so that on short circuit the current is little different from that at maximum P.D.

Now, armature-reaction is proportional to armature AT for a given field, but there is a limit to the armature-reaction possible with good commutation. We may thus say that—

(1) The number of commutator segments must be as large as will keep the reactance-voltage (V_r) down to about two volts per segment.
(2) With the current at which the machine is supposed to run, the armature-reaction should be sufficient to distort the field very considerably, *i.e.* the ratio—

$$\frac{\text{Armature AT}}{\text{field AT}} \text{ should approach } \frac{1\cdot5 \text{ or } 2}{1}$$

The effect of armature-reaction is more marked if the brushes are given a greater lead, *i.e.* if the back AT are increased. This makes the curve much steeper, but at the same time decreases the maximum P.D. obtainable, as the brushes are not in the position of maximum E.M.F.

Movement of Brushes.—The machine can be made self-regulating if the curve is steep to start with, and if as the number of lamps decreases the brushes are automatically moved forward. This is the principle of the patent of the "Thomson-Houston arc-lighting generator."

The above is an outline of the design of constant-current series generators. The subject is too special and limited to require further treatment here, but attention may be directed to the paper on the subject by Prof. Wilson before the I.E.E.

Series Motors.—For a motor the only part of the characteristic used is from A to B (Fig. 124). An increase in load means an increase in the field, and as torque is proportional to the product of field and of armature AT, the torque varies as the square of the current up to the point of saturation, and then approximately as the current. Now, for a shunt motor the torque varies almost as the armature current. Hence a series motor is used where for a given armature current a large torque is required, and where a large range of speed is desired.

Series motors are used for two purposes—

(1) Traction.

(2) Lift and crane work.

For lift work, however, to prevent excessive speed, a compound motor is often preferred. The tendency is thus to design series motors for traction, and to fit them in for crane work if desired.

Traction Motors.—For traction work the following points have to be remembered :—

(1) The available space is very small.

(2) It is important to keep the weight of the machine down, as it has to be moved about with the car.

(3) The motor must be designed so that it will stand exceptionally heavy strains.

In railway work the motor armature is placed directly on the wheel axle.

For tramways three systems have been used—

(1) Armature directly coupled to axle.

(2) Armature with single gearing.

(3) Armature with double gearing.

Type (2) is practically the only one now adopted. Type (3) was found to be noisy, inefficient, and difficult to maintain, whereas (1) required a large size of motor, and vibration was transmitted to the machine from the road.

The size of motor used for average tramcars is from 20 to 30 H.P., and for single gearing the ratio is about 5 to 1, the number of teeth in the pinion being about 14, and in the wheel 67 to 68.

The pinion is machine cut, and made of hard steel.

The wheel is also machine cut, but of soft steel.

Width of pinion, 4″ to 5″.

Efficiency, about 80 per cent. with the gearing (see Fig. 19), and maximum efficiency is at $\frac{1}{2}$ to $\frac{3}{4}$ full load.

The weight of a 25 H.P. motor is about 1500 lbs. complete with the case.

Problem III.

27-H.P. Series Motor.

Limiting Dimensions.—The dimensions are settled to some extent by the size of the car. A 25 to 30 H.P. motor will have to run at such a speed that when driving a car wheel of 30″ diameter, the speed at full load will be about 10 miles per hour.

Taking the gear ratio 4·8, this gives a motor speed at full load of 550 to 600 r.p.m. The speed is thus fixed. The maximum current is decided by the voltage of the system, and taking this at 500 volts, full load current = 50 amps.

The general proportions, densities, etc., are not different from those adopted in ordinary practice as discussed in Chapters II. and III.; but there are certain conditions, notably those of small weight and short rating, which to some extent modify the ordinary values. The following are some usual figures :—

Density at the pole-face .	50,000 to 70,000 lines per sq. in.
Density in the yoke . .	90,000 lines per sq. in.
Leakage factor about . .	1·25
Number of poles . . .	4

Type of Winding.—As all the surface of the commutator cannot be seen, the winding will conveniently be two-circuit with two brushes at 90°, preferably arranged at the top of the commutator.

Poles.—These are invariably laminated, and sometimes space blocks are used to form ventilating ducts parallel with and opposite to those in the armature.

Temperature Rise.—Traction motors are always totally enclosed, and usually rated at their full load for one hour with a rise of temperature not exceeding 55° C. on any part. Since the efficiency is about 80% (see Fig. 19), of which about 5% is lost in the gear, we shall have the following preliminary values of losses for a 27 H.P. motor :—

Output . . .	20,000 watts on second motion shaft.
Input	25,000 watts or 50 amps. at 500 volts.
Loss in gear 5% . .	1250 watts.
Other losses . .	3750 watts.

The latter must be dissipated as heat by the motor.

The input is about 42 watts per revolution per minute, which from curves like those of Fig. 49, when the higher temperature has been allowed for, corresponds to about 1 sq. in. of external surface for each watt dissipated. About 3700 sq. ins. of external surface are thus seen to be necessary. From trial designs, or by reckoning out the external surface corresponding to such motors as are illustrated in Example I., it is soon seen that this value corresponds, in the case of a four-pole machine, to an armature diameter of from 12″ to 14″. Accordingly we find in practice that traction motor armatures are usually about this diameter. It would be convenient in many cases to make the armature larger in diameter than the above, but the standard 30″ wheel leaves then but little clearance between the bottom of the motor and the road, unless the centre line of the motor is brought very much above the wheel axle, when the yoke tends to foul the car floor.

Commutation Limits.—It has already been shown in connection with Problem I. that a reasonable value of V_r without interpoles corresponds to a maximum of five turns per section. Again, in Problem IV. an instance is given of the connection between flux per pole and turns per section for constant V_r. Applying the latter to this case, or arguing from the former, leads to the conclusion that 4 turns per section would be a large number to adopt, and three turns per section would lead to a value of V_r quite high enough. In spite of this, up to the present four turns per section have been mostly used, though commutation is, in the author's opinion, unsatisfactory with this number. Three turns per section leads to a machine rather more costly, but unless interpoles be

universally adopted this type of armature will certainly be more generally used, as the commutator lasts much longer.

Division of Losses.—The various losses cannot be treated quite in the same way as for constant-speed machines, because all the losses are

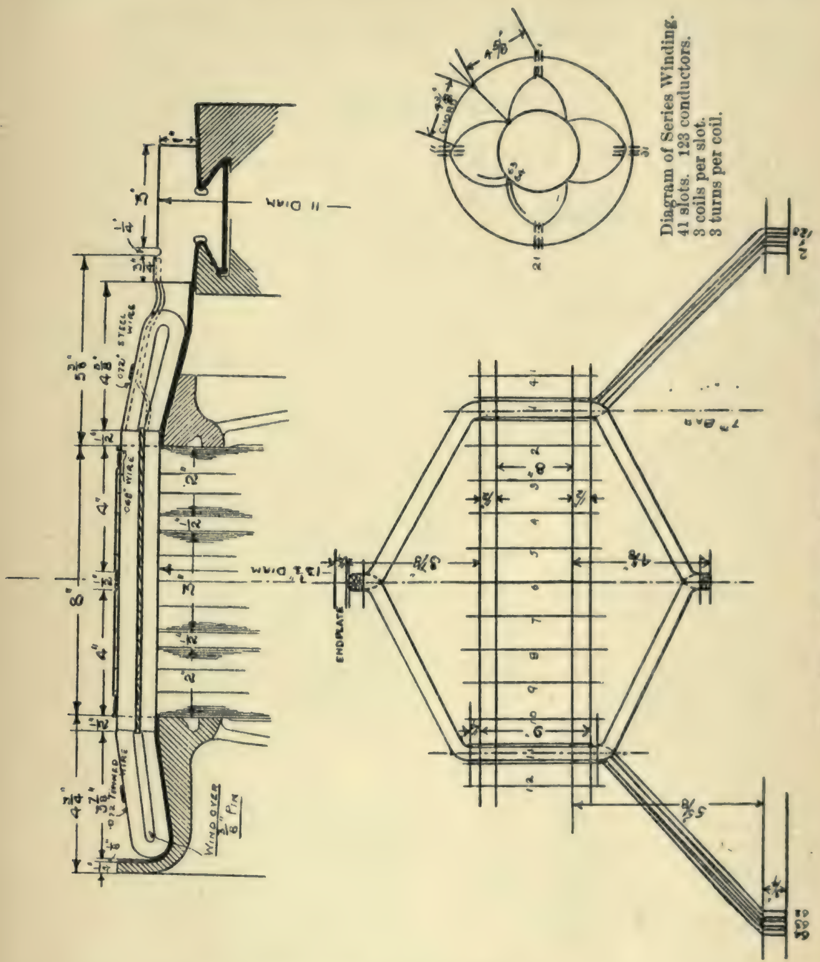


FIG. 129.—TRACTION MOTOR. ARMATURE DETAILS.

now more or less variable. The iron loss varies both with the speed and current, and since the speed rises when the current falls, probably the iron loss remains more or less constant. The sum of the friction and iron losses rises as the speed falls, but not very rapidly. Thus in the case of such a motor tested at the Municipal School of Technology, Manchester,

the sum of the iron and friction losses only varied from 780 watts to 900 watts, while the speed changed from 500 to 200 r.p.m.

Table XXVIII. and Figs. 129, 130, and 131 give particulars of a crane or traction motor for 500 volts, designed on the principles outlined above, with three turns per commutator section. The student will find it a good exercise to start from first principles, as was done in the case of Problems I. and II., and work towards the dimensions given.

TABLE XXVIII.

Type of Machine	Series Motor
H.P.	27
Revolutions per minute	600
Poles	4
Armature—	
Armature diameter	13·5″
Core-length	8″
Pole-pitch	10·6″
Pole-arc	7·5″
Pole-face area	60 sq. ins.
Flux issuing from shoe	3×10^6
Generated volts at full load	450
Number of slots	41
Coils per slot	3
Turns per coil	3
Length of mean turn	40″
Armature conductors	738
Size of conductor	No. 13 S.W.G.
Slot-depth	$1\frac{3}{16}$″
Slot-width	0·5″
Slot arrangement	as Fig. 130
Armature resistance	0·4 ohm hot (70° C.) nearly
Tooth-root density	164,000 (uncorrected)
Radial depth of stamping below teeth	$2\frac{3}{8}$″
Stamping arrangement	Fig. 102
Density below teeth	100,000
Diameter of shaft in centre	$3\frac{1}{4}$″
Diameter at gearing journal	$2\frac{7}{8}$″
Over-all length of armature	16″
Armature arrangement	Fig. 129

Binding Wires.—On the core there will be three bands of binding wire (tinned steel), each band consisting of about 12 turns of No. 18 S.W.G. On the end-connections at each end there will be one band consisting of about 18 turns of No. 16 S.W.G.

Armature Core-heads or End-plates.—These are partly shown in Fig. 129. They are made in malleable iron.

Commutator (see Fig. 129)—

No. of segments	123
Thickness of segment (max.) .	0·25″
Insulation thickness . . .	0·03″
Current density under brush .	40 amps. per sq. in.
Brushes per spindle . . .	2
Axial length of brush . . .	2″
Thickness of brush . . .	$\frac{3}{4}$″
Coils short circuited at each brush	3

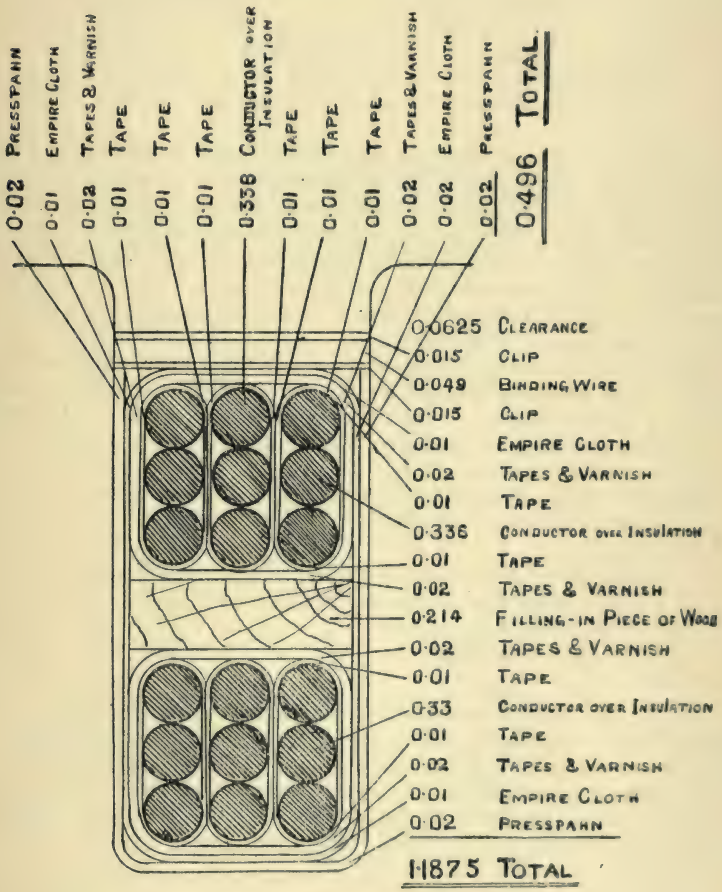


Fig. 130.—Armature Slot Details.

Where the commutator bush fits the shaft, the latter should be

slightly tapered and the bush fixed on by a key. The bush is generally solid cast iron (no spider).

The depth of segment allowed for wear varies, being often stated in the specification; it generally lies between $\frac{1}{2}''$ and $1''$ ($1''$ is excessive); take $\frac{3}{4}''$. This gives $1''$ from top of segment to the top of the bush. The thickness of the end rings (which are of mica) is rarely less than $\frac{3}{32}''$, and the ends of these rings are bound down with string.

Field-magnet.—The length of the air-gap varies with different makers, being often specified when the motors are ordered. As a maximum $\frac{1}{4}''$ is used, but for A.C. motors it may be as small as $\frac{1}{8}''$. In the present design we take $\frac{3}{16}''$.

Density in pole . . .	100,000
Flux per pole . . .	$3 \times 10^6 \times 1{\cdot}25 = 3{\cdot}75 \times 10^6$
Section of pole-core . .	37·5 sq. ins.

If the axial length of the pole-core be the same as that of the armature, and if these cores have space blocks to allow of an air circulation in the pole itself, and to reduce eddy-currents therein, then the net axial length of the pole is the same as that of the armature, viz. 6·3″. The width of the pole is then 6″. This, however, is in the author's opinion unnecessary. It is desirable that some space blocks be used,* and if two $\frac{1}{4}''$ each are adopted in place of two of $\frac{1}{2}''$ each, the net axial length of the pole is 6·8″ and its width 5·5″.

Yoke Dimensions.—It is not, as a rule, advisable to consider the length of yoke carrying flux as much greater than the armature gross length, say in this case 10″. Thus fixing upon a density of, say, 90,000 for the yoke (it being of good steel), we get the necessary area as 20·7 sq. in., and the thickness as 2″.

Losses.—The armature iron-loss at full load when estimated by the rough formula (p. 29) is 720 watts.

Bearing friction estimated at 1% . .	220 watts
Commutator resistance loss (cf. Fig. 83)	130 ,,
Commutator friction loss . . .	42 ,,
Armature resistance loss . . .	1000 ,,

The sum of the above losses is 2112, which subtracted from 3750 leaves roughly 1600 watts for the series field and various losses not calculated, such as eddy-currents in the pole-shoes, etc. If we calculate the field for 1500 watts, the losses should be on the safe side.

Field Ampere-turns.—The following table gives the calculated values of field ampere-turns at full load :—

* Otherwise with such wide armature ducts flux tends to pass down the teeth flanks at each duct, setting up thereby considerable eddy-currents even in the space-blocks of the armature.

Part.	Material.	Density.	Length.	Amp. turns per inch.	Amp. turns.
Armature core	wrought iron (special)	100,000	3·4″	30	102
Teeth . . .	” ”	154,000 (mean)	1·1875	2,600	3000
Gap	air	54,000	0·1875	17,000	3180
Pole	wrought iron	100,000	$3\frac{1}{8}$	30	94
Yoke . . .	cast iron	90,000	10	30	300
				Total .	6676

To the above must be added an allowance for armature-reaction. Since the motor has to be reversible, the brushes must short-circuit coils lying midway between the pole-tips, *i.e.* there will be no back ampere-turns. The distorting (or cross) armature ampere-turns with a current of 50 amps. will be 2300. The constant from Fig. 39 is 0·34. Therefore

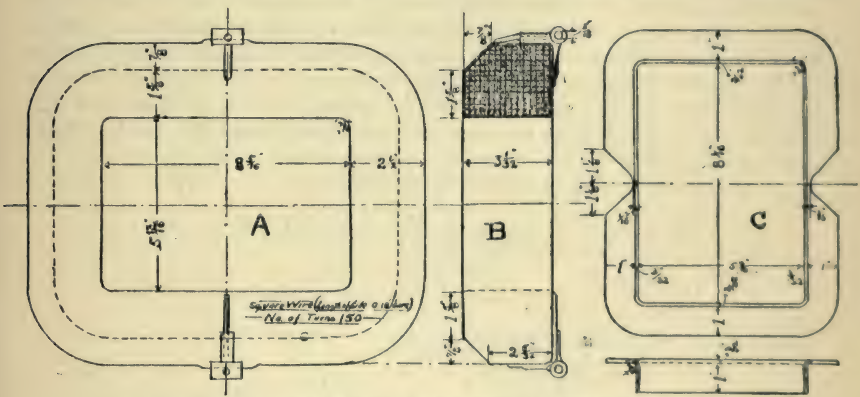


FIG. 131.—TRACTION MOTOR FIELD-COIL. SKETCH OF DETAILS.

the compensating ampere-turns are 780, so that the total ampere-turns to be provided per pole are in round numbers 7500.

Field Coil.—With a current of 50 amps. 150 turns per coil will be needed, and for 1500 watts lost the resistance per coil must be 0·15 ohm. Fig. 131 shows the least room that will be taken up by a coil consisting of 150 turns of wire of square section of 0·18″ side, having a resistance of 0·144 ohm. In this diagram A and B show the completed coil, and C is the half-spool used to support and fix the coil. For traction purposes, where space is of great importance, such a coil would be of advantage, but for lift or crane work it would be prefer able to adopt a coil consisting of

150 turns of say No. 5 S.W.G. with a larger, longer pole. The space-factor in the latter case would be worse and the machine larger. But these disadvantages would usually be more than compensated for by the greater ease of winding. Square and rectangular wires are very difficult to keep flat and always tend to cut the insulation.

Problem IV.

The preceding problems have all dealt with the derivation of new machines. Often, however, the designer is faced with the question of making the best of an old design to suit new conditions. Very often it is a matter of altering a machine designed for a voltage such as 230, so that it may be suitable for 400 or 500 volts. Such a case is the following :—

A standard four-pole 220-*volt machine, having an armature-core* $12\frac{3}{4}''$ *diameter* $\times$ 8″ *long with* 75 *slots and* 75 *commutator bars, is required to be remodelled so as to be suitable for* 16 K.W. 500 *volts, and* 400 *r.p.m., with a reactance-voltage* (V_r) *not exceeding three. The original flux was* $2\frac{1}{2}$ *million lines per pole, which should not be exceeded, or alteration of yoke and pole will be necessary. Give the minimum number of commutator segments that will be required for satisfactory operation without substantial alteration to the flux.*

Now, we know, to begin with, that the original number of slots is inconveniently large, and that some number in the neighbourhood of 39 would be more economical. This only means that there must be a change in the number of coils per slot; it gives no clue to the number of turns per coil. To ascertain the limits of the latter, we first substitute in that formula for V_r which is in terms of the watts, poles, and flux per pole, and is given on p. 128.

In this formula the values are as follows :—

$$\begin{aligned} EC &= 16{,}000 \text{ watts} \\ \text{Net armature length} &= 6'' \\ \text{Pole-pitch} &= 10'' \\ V_r &= 3 \text{ volts} \end{aligned}$$

consequently we obtain the relationship—

$$\text{Flux} = 4 \times 10^5 \times (\text{t.p.s.})$$

Taking now the E.M.F. formula, and assuming that the maximum voltage to be generated at the normal speed of 400 r.p.m. is 460 volts, we have—

$$E = \text{flux} \times \text{conductors} \times n \times 2 \times 10^{-8} \text{ for a two-circuit winding}$$

Substituting for the values of voltage and speed, we get—

$$\text{Flux} \times \text{conductors} = 345 \times 10^7$$

But flux $= 4 \times 10^5 \times$ (t.p.s.), from the preceding formula, and conductors $=$ (t.p.s.) $\times 2s$, where s is the number of sections, so that—

$$s(\text{t.p.s.})^2 = 4312$$

We now tabulate as follows :—

Sections.	Turns per section.	Flux per pole.
141	5·5	$2 \cdot 2 \times 10^6$
123	5·9	$2 \cdot 36 \times 10^6$
111	6·2	$2 \cdot 8 \times 10^6$

It is then clear that without increasing the original flux, the maximum possible number of turns per section at this speed is 6, and that this number corresponds to 123 sections.

Thus the machine reaches its limit, as far as reactance-voltage is concerned, when arranged with 41 slots, 3 coils per slot (*i.e.* 6 half-coils), 6 turns per coil.

Problem V.

Of a similar character to the last problem is the question that arises when it is necessary to determine the best radial depth of iron below the teeth. This is of importance up to the point of saturation, because, though the total ampere-turns are hardly affected, yet the less the radial depth the better is the internal ventilation ; and it is therefore important to know how the iron losses, which increase with the density, will be affected by the change in radial depth. To make the question more clear, consider the following general case :—

A four-pole machine having an armature D_1'' diameter at the tooth roots, with gross length l'' and net length l_1'', has to carry N lines per pole. What will be the best radial depth of stamping from the point of view of temperature-rise and efficiency, assuming the iron-loss formula of p. 29 as correct?

Let D_i be the internal diameter of the stampings, then—

$$\text{Vol. of iron in armature below the teeth} = \frac{\pi}{4}\left(D_1^2 - D_i^2\right)l_1$$

$$\text{Sectional area of iron in the armature} = \frac{D_1 - D_i}{2} l_1$$

$$\text{Corresponding density} = \frac{2N}{2(D_1 - D_i)l_1}$$

The frequency is constant, so that—

$$\text{Iron-loss, neglecting teeth} = (D_1 + D_i) \times \text{constant}$$

i.e. the smaller D_i the better is the efficiency.

Now, from Fig. 46 the cooling surface is proportional to—

$$\pi DL_1 + \frac{\pi}{4}(D^2 - D_i^2)(n_d + 2) + \pi D_i l$$

where n_d is the number of ducts, and L_1 is the length of the armature including end connections.

Differentiating this with respect to D_i, we get—

$$-\frac{\pi}{2}D_i(n_d + 2) + \pi l$$

and the cooling is therefore a maximum when—

$$\tfrac{1}{2}D_i(n_d + 2) = l$$

Thus the answer to the question is that the armature losses increase with an increase of the hole D_i, but so also does the cooling surface up to the point when

$$D_i(n_d + 2) = 2l$$

The best density will therefore depend upon the particular machine. The fact that the teeth have been neglected does not affect the argument, provided that the slot depth is a constant fraction of D, as it will be by the formula of p. 15. The best density in any particular case can easily be deduced from the above formulæ; it will evidently depend on the frequency, iron-loss constant, number of ventilating ducts, and the constant chosen in the temperature-rise formula (method 2), and it is always subject to the limitations of saturation, and the proportion of D_i occupied by the shaft.

As a particular instance, let $D = 13{\cdot}5''$, $D_1 = 11''$, $L_1 = 17''$, $l = 8''$, $l_1 = 6{\cdot}3''$, $\sim = 20$, $n_d = 2$, $N = 3 \times 10^6$. Iron loss constant = 1·8. These data correspond almost with Fig. 129.

Then iron losses due to iron below teeth $= 261 + 24D_i$.

Cooling surface is a maximum when $D_i = 4{\cdot}3''$.

This results in a density of about 68,000 lines below the teeth. When the shaft, however, is considered, this would leave too little passage for air, so that practice modifies the results to some extent, as is seen by reference to Fig. 102.

APPENDIX I.

Relationship between Depth of Slot, Diameter of Armature, and Magnetic Densities.

Let w_s = width of slot;
w_t = width of tooth at the armature periphery;
t = number of teeth.

Then at the armature periphery—

$$w_s + w_t + \pi D/t.$$

Let $w_s = m_1 \, . \, w_t$, so that m_1 is a ratio and
$w_t = \pi D/(1 + m_1)t$; $w_s = \pi D m_1/(1 + m_1)t$.

Let D_b = the diameter of the armature measured at the bottom of the slots, then—

$$\text{Width of tooth-root} = \pi D_b/t - \pi D m_1/(1 + m_1)t$$
$$= \frac{\pi}{t}\left\{\frac{D_b(1 + m_1) - m_1)D}{1 + m_1}\right\}$$

Let β_t = average density, at the roots of the teeth;
l = net iron length of the armature core.

Then—

$$\text{Flux per tooth} = \beta_t l \frac{\pi}{t}\left\{\frac{D_b(1 + m_1) - m_1 D}{1 + m_1}\right\} \quad . \quad . \quad (1)$$

Now let r_p = ratio of pole-arc to pole-pitch; then—

$$\text{Pole-face area} = L \, . \, \pi D \, . \, r_p/p$$

Let B_f be the average density at the pole-face; then—

$$\text{Flux issuing from the pole-face} = \beta_p L \pi D r_p/p \quad . \quad . \quad (2)$$

Now, the number of teeth carrying the flux is $t \, . \, r_p m_2/p$, where m_2 is a ratio greater than unity to allow for the "fringing" which takes place at the pole-tips.

Multiplying (1) by the number of teeth gives us the total flux carried by the teeth per pole; its value is

$$\beta_t l \frac{\pi}{p}\left\{\frac{D \, . \, (1 + m_1) - m_1 D}{1 + m_1}\right\} r_p m_2 \quad . \quad . \quad . \quad . \quad (3)$$

Evidently this flux must be the same as that given by (2).

Thus—

$$\frac{\beta_p LD}{\beta_t l m_2} = \frac{D_b(1+m_1) - m_1 D}{1+m_1}$$

Putting $\dfrac{\beta_p L}{\beta_t l m_2} = K$, and transposing—

$$KD(1+m_1) = D_b(1+m_1) - m_1 D$$

$$\text{or } \frac{D}{D_b} = \frac{1+m_1}{K(1+m_1)+m_1}$$

$$\text{Whence depth of slot } \left(= \frac{D - D_b}{2}\right) = \frac{1 - K(1+m_1)}{2(1+m_1)} D$$

In order that this may be positive,

$$1 \text{ must be} > K(1+m_1)$$

$$\text{or } K. < \frac{1}{1+m_1}$$

As an example, if $\beta_p/\beta_t = 1/2{\cdot}7$, $l/L = 0{\cdot}8$, $m_2 = 1{\cdot}1$, $m_1 = 1$; then—

$$K = 0{\cdot}42 \text{ and}$$

$$\text{depth of slot} = 0{\cdot}04D$$

APPENDIX II.

The Connection between the Efficiency of the Machine and the Resistances of the Circuit.

In any electric generator the electrical efficiency is given by the expression—

$$\text{Electrical efficiency} = \frac{\text{watts output}}{\text{watts output} + \text{electrical losses}}$$

In the case of a series-wound machine this is—

$$\text{Electrical efficiency} = EC/\{EC + C^2(R_a + R_f)\} = E/\{E + C(R_a + R_f)\}$$

From which it is clear that the electrical efficiency is a maximum when $(R_a + R_f)$ is a minimum.

In the case of a shunt-wound generator, however, there is a particular relationship between the various resistances in the circuit for which the electrical efficiency is a maximum.

Let η_1 be the electrical efficiency of the machine; then—

$$\eta_1 = \frac{\text{watts output}}{\text{watts output} + \text{losses}} = \frac{EC}{EC + \left(C + \dfrac{E}{R_f}\right)^2 R_a + \left(\dfrac{E}{R_f}\right)^2 R_f}$$

Now, let R be the external resistance corresponding to the current C.

Then $C = E/R$, and

$$\eta_1 = \frac{E^2}{R} \div \left\{\frac{E^2}{R} + \left(\frac{E}{R} + \frac{E}{R_f}\right)^2 R_a + \left(\frac{E}{R_f}\right)^2 R_f\right\}$$

$$= \frac{1}{1 + \frac{RR_a}{R_f^2} + \frac{R_a}{R} + \frac{R}{R_f} + \frac{2R_a}{R_f}} = \frac{1}{M} \text{ say} \quad . \quad . \quad . \quad (1)$$

Now, when η_1 is a maximum it is clear that the denominator (M) in the foregoing expression must be a minimum. Differentiating M with respect to R and equating to zero, we have—

$$\frac{dM}{dR} = -\frac{R_a}{R^2} + \frac{R_a}{R_f^2} + \frac{1}{R_f} = 0 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\frac{d^2M}{dR^2} = \frac{2R_a}{R^3},$$

which is positive, so that (2) will be a minimum.

$$\text{From (2) we get } R = \sqrt{\frac{R_aR_f^2}{R_a + R_f}} = R_f\sqrt{\frac{R_a}{R_a + R_f}}. \quad (3)$$

[Usually R_a is small compared with R_f, so that η_1 is a maximum when—

$$R = R_f\sqrt{\frac{R_a}{R_f}}$$

or $R = \sqrt{R_aR_f}$ nearly.]

In order to find the value of η_1 when it is a maximum, we substitute (3) in the denominator M, thus—

$$M \text{ (minimum)} = 1 + 2\frac{R_a}{R_f} + \frac{2R_a}{R_f\sqrt{\frac{R_a}{R_a + R_f}}}$$

whence $$\eta_1(\max) = 1 \div \left\{1 + 2\frac{R_a}{R_f} + 2\frac{R_a}{R_f}\sqrt{1 + \frac{R_f}{R_a}}\right\}$$

This leads to the quadratic equation—

$$\eta_1^2 - 4\eta_1\frac{R_a}{R_f} - 2\eta_1 + 1 = 0$$

with the solution $$R_a = R_f\frac{(1 - \eta_1)^2}{4\eta_1}$$

And substituting the value of R_a in the equation—

$$R = R_f\sqrt{\frac{R_a}{R_a + R_f}}$$

$$\text{gives } R_f = R\frac{1 + \eta_1}{1 - \eta_1}$$

$$\text{and } R_a = R\frac{1 - \eta_1^2}{4\eta_1}$$

These values correspond to the maximum point of the electrical efficiency curve.

In compound-wound machines two cases arise. For the long-shunt type the above relationships are true if R_a be held to include the resistance of the series coils. For the short-shunt type a slight but practically negligible change is introduced owing to the compound loss being proportional to C^2 instead of $\left(C + \frac{E}{R_f}\right)^2$.

The electrical efficiency η_1 is not always easy to assume or derive; but the application of these relationships to η, the commercial efficiency, is given on p. 27.

APPENDIX III.

Calculation of Leakage Flux.

On pp. 42 and 43 the main considerations governing the calculation of leakage flux have been mentioned. The amount of flux leaking from one surface to another being difficult to estimate, it is useful to have at hand a few standard cases founded on broad assumptions which yield useful approximate values. The six cases given below (the first three of which are due to Professor Forbes) are examples of such useful standards. In each case it is assumed, for the reasons given on p. 43, that the reluctance of the iron part of the leakage path may be neglected. The distribution of flux forming the basis of the calculations is, of course, far from being exactly in accordance with that which actually exists, but it yields values for the flux which are found to be sufficiently accurate for practical purposes.

Case I.—Leakage between two parallel surfaces (Fig. 132) *of areas* A_1 *and* A_2 respectively. The lines of induction are assumed to be straight, as indicated in the figure.

$$\text{Flux} = \text{ampere-turns} \times \frac{1}{2} \cdot \frac{A_1 + A_2}{l \times 0{\cdot}313}$$

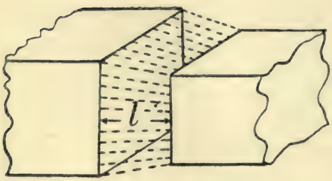


Fig. 132.—Leakage Flux. Case I.

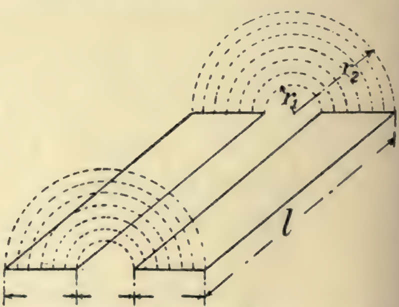


Fig. 133.—Leakage Flux. Case II.

Case II.—Leakage between two parallel surfaces in approximately the

same plane, separated by a short gap (Fig. 133). The lines of induction are assumed to be semicircles as indicated.

$$\text{Flux} = \frac{\text{ampere-turns}}{0{\cdot}313} \times l \int_{r_1}^{r_2} \frac{1}{\pi r} dr$$

$$= \frac{\text{ampere-turns}}{0{\cdot}313} \times 2{\cdot}3 \frac{l}{\pi} \log_{10} \frac{r_2}{r_1}$$

Case III.—Leakage between two parallel surfaces in approximately the same plane separated by a comparatively long gap (Fig. 134). The lines of induction are assumed to be straight, with quadrants at the ends as indicated.

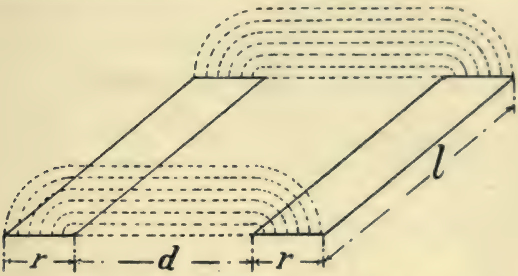


FIG. 134.—LEAKAGE FLUX. CASE III.

$$\text{Flux} = \frac{\text{ampere-turns}}{0{\cdot}313} l \int_{0}^{r} \frac{1}{d + \pi r} dr$$

$$= \frac{\text{ampere-turns}}{0{\cdot}313} 2{\cdot}3 \frac{l}{\pi} \log_{10} \frac{\pi r + d}{d}$$

NOTE.—Obviously, if the surfaces are not parallel, but are inclined to one another at an angle θ, the circular measure of this angle is to be substituted for π in both Case II. and Case III.

Case IV.—Leakage between parallel surfaces (or slots), with electric

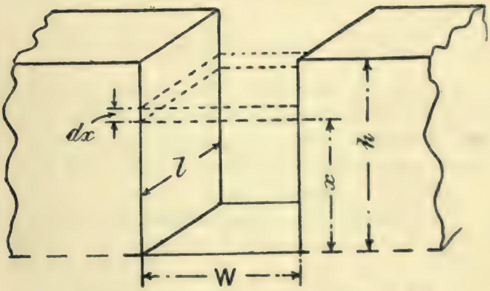


FIG. 135.—LEAKAGE FLUX. CASE IV.

conductors passing between.—In Fig. 135 let the slot shown be supposed full

of conductors, t in number, each carrying a amperes. Then the flux across any section ldx is due to $\frac{tax}{h}$ ampere-conductors, and may be written

$$\text{Flux through } dx = \frac{lxta}{0{\cdot}313\mathrm{W}h}dx$$

$$\text{Total flux across the slot} = \int_0^h \frac{lta}{0{\cdot}313h\mathrm{W}}xdx$$

$$= \frac{l}{0{\cdot}313} \times \text{ampere-conductors} \times \frac{1}{2} \cdot \frac{\text{depth of slot}}{\text{width of slot}}$$

If the slot is not completely filled, but a space is left empty, this space can be treated under Case I. Leakage from teeth to neighbouring iron can be taken by Carter's curve (see p. 47).

Case V.—Leakage between surfaces projecting from a yoke and inclined to one another at an angle θ, with electric conductors carrying current evenly distributed down them.—This case is that which occurs in the field magnets of a multipolar machine, and is illustrated in Fig. 136. The

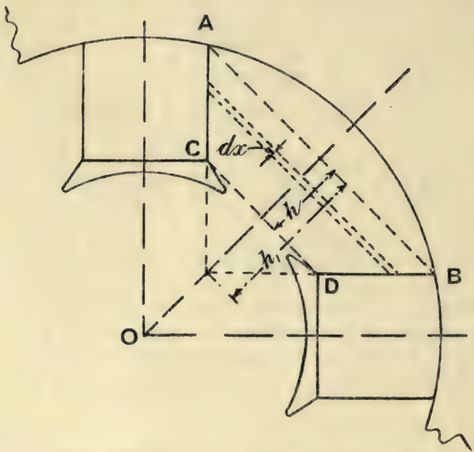


FIG. 136.—LEAKAGE FLUX. CASE V.

flux may be assumed as passing from surface to surface, following concentric circular paths between the planes AB and CD, or it may be supposed to pass straight across in parallel lines from one side to the other. The latter supposition yields results more nearly approaching those taken from practical measurement.

Let $t.a.$ be the total ampere-conductors lying between the pair of surfaces, and let W be the distance AB, and h be the projected length AC. Then—

$$\text{The flux along a strip } dx = \frac{t.a.l}{0{\cdot}313h} \times \frac{x}{\mathrm{W} - 2x\tan\frac{\theta}{2}}dx$$

$$\text{Total flux} = \frac{t.a.l}{0{\cdot}313h}\int_0^h \frac{x}{W - 2x\tan\frac{\theta}{2}}dx$$

$$= \frac{t.a.l}{0{\cdot}313h}\left(-\frac{h}{2\tan\frac{\theta}{2}} + \frac{2{\cdot}3W}{4\tan^2\frac{\theta}{2}}\log_{10}\frac{W}{W - 2h\tan\frac{\theta}{2}}\right)$$

Thus for a 4-pole machine $\tan\frac{\theta}{2} = 1$, and the leakage flux is—

$$\frac{t.a.l}{0{\cdot}313h}\left(-\frac{h}{2} + \frac{2{\cdot}3W}{4}\log_{10}\frac{W}{W - 2h}\right)$$

Case VI.—Flank leakage. In a machine with poles of rectangular section, to the leakage between pole and pole as in Fig. 136, must be added that which takes place between the flanks or side-faces of the

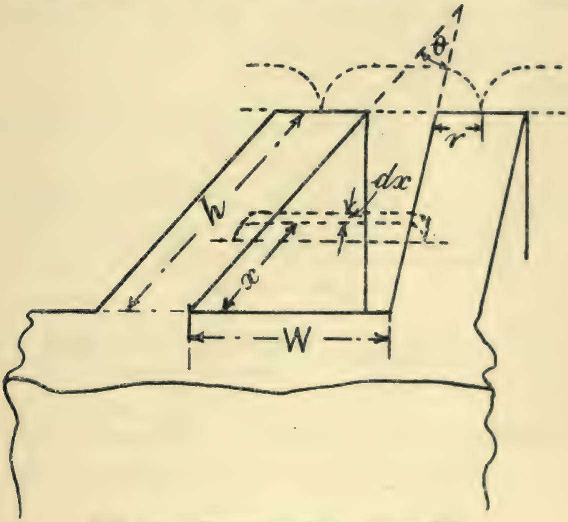


FIG. 137.—LEAKAGE FLUX. CASE VI.

poles as suggested in Fig. 137. This leakage may be estimated by a method combining Case III. with Case V. If we assume that each projection in Fig. 137 is wound from end to end, so that acting across the space between each pair there is the magneto-motive force due to the ampere-turns on the pair, and if the lines of induction be supposed straight in the middle with quadrants at the ends as indicated in the figure, then—

Leakage flux from one flank to the next

$$= \int_{x=0}^{x=h}\int_{r=0}^{r=r}\frac{\text{ampere-turns}}{0{\cdot}313h}\cdot\frac{x}{W - 2\tau x + \pi r}drdx$$

$$= \frac{\text{ampere-turns}}{0{\cdot}313h\pi}\left[\frac{2{\cdot}3}{2}h^2\log_{10}\frac{W - 2\tau h + \pi r}{W - 2\tau h} + \frac{1}{4\tau^2}\left\{\frac{2{\cdot}3W^2}{2}\log_{10}\frac{W - 2\tau h}{W}\right.\right.$$

$$\left.\left. - \frac{2{\cdot}3(W + \pi r)^2}{2}\log_{10}\frac{W - 2\tau h + \pi r}{W + \pi r}\right\} - \frac{\pi r h}{4\tau}\right]$$

in which $\tau = \tan\frac{\theta}{2}$ (= 1 for a four-pole machine).

APPENDIX IV.

Relationship between Length and Depth of Field-Coil (l_c and d_c), Fig. 44.

The resistance of any coil is—

$$\frac{\text{(specific resistance of copper)} \times \text{turns} \times \text{length of mean turn}}{\text{area of wire}}$$

The fall of potential across the coil for a given field current is—

$$\frac{\text{(spec. res.)} \times \text{amp.-turns} \times \text{length mean turn}}{(\text{dia. wire})^2 \times \frac{\pi}{4}} \quad . \quad . \quad (1)$$

$$\text{Now, turns per coil} = \frac{\text{length of coil} \times \text{depth of coil}^*}{(\text{dia. wire} + \text{insulation})^2}$$

We may write—

diameter of wire + insulation = m(diameter of wire)

where m is a multiplier which will vary with each wire. Thus—

$$\frac{\text{ampere-turns per coil}}{\text{field current}} = \frac{l_c \times d_c}{m^2 \times (\text{dia. wire})^2} \quad . \quad . \quad . \quad (2)$$

Substituting the value for the diameter of the wire as obtained from (2) in (1), we obtain—

$$l_c d_c = \frac{4 \text{ spec. res.} \times (\text{amp.-turns per coil})^2 \times \text{length mean turn} \times m^2}{\pi \text{ watts per coil}}$$

The value of the specific resistance should be that corresponding to the mean temperature of the coil. Say 82×10^{-8}.

* There is no allowance for bedding.

APPENDIX V.

Theory of Pure E.M.F. Commutation.

In any closed circuit the sum of the P.D.'s is zero. Now, in the short-circuited coils under the brush there are three P.D.'s acting, viz.—

(1) The P.D. due to the current passing through a circuit having resistance: if C_i be the instantaneous value of the current at a time t seconds after the commencement of commutation, this P.D. is $C_i r$, where r represents the resistance in the circuit at the time t. This resistance is made up of the resistance of the brush, the brush contacts and the coil. Since we are now dealing with pure E.M.F. commutation, we shall allow nothing for the changing area of brush contact, so that r becomes constant and may be taken to be the resistance of the coil plus the least resistance the brush ever offers. The latter may be very small, for with proper E.M.F. commutation, such as is secured with interpoles, there should be no necessity for high-resistance brushes.

(2) The P.D. due to self-induction, *i.e.* due to the changing magnetic flux set up by the current itself. If L_c be the coefficient of self-induction of the coil, this P.D. has a value $= L_c \dfrac{dC_i}{dt}$.

(3) The P.D. set up in the coil due to its rotation through the magnetic field under the interpole, or pole-tip. This we will call e.

Thus we have—

$$C_i r + L_c \frac{dC_i}{dt} + e = 0$$

or

$$r\left(C_i + \frac{e}{r}\right) = -L_c \frac{dC_i}{dt}$$

$$-\frac{r}{L_c}dt = \frac{r}{C_i r + e}dC_i$$

Integrating

$$-\frac{r}{L_c}t = \log_e \frac{C_i r + e}{r} - \log_e A$$

where A is the integration constant.

Now, at the beginning of commutation, *i.e.* at time $t = 0$—

$$C_i = \text{current per conductor} = C_w$$

or

$$A = \frac{C_w r + e}{r}$$

Thus

$$-\frac{r}{L_c}t = \log_e \frac{C_i r + e}{C_w r + e}$$

At the end of commutation, when $t = t_c$, C_i should have a value $= -C_w$,

if the brush is to leave the segment without spark. Under these conditions—

$$-\frac{r}{L_c}t_c = \log_e \frac{e - C_w r}{e + C_w r}$$

or

$$\epsilon^{-\frac{r}{L_c}t_c} = \frac{e - C_w r}{e + C_w r}$$

i.e.

$$e = C_w r \frac{1 + \epsilon^{-\frac{r}{L_c}t_c}}{1 - \epsilon^{-\frac{r}{L_c}t_c}}$$

This really shows the value of e necessary at the end of commutation, *i.e.* at the critical moment when a segment leaves the brush. It is, however, sufficient for interpole calculations to proportion the interpole and its windings so that an E.M.F. at least equal to e may be introduced.

APPENDIX VI.

Armature-Dimensions and Reactance-Voltage.

From p. 127 we have—

$$V_r = \frac{2C_w Sn(40l + 6pp)(tps)^2}{10^8} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now, $l = 0{\cdot}8d$ nearly (p. 20).

Consider first the case of circular poles. We have from p. 20—

$$d = 1{\cdot}5\lambda D/p$$

Suppose the average value of λ to be $1{\cdot}15$;

$$\text{then } D = pd/1{\cdot}73$$

$$\text{Also the pole-pitch (p.p.)} = \pi D/p = \pi d/1{\cdot}73$$

$$\text{so that } 40l + 6pp = 32d + 10{\cdot}9d = 43d \quad . \quad . \quad . \quad (2)$$

$$\text{whence } V_r = \frac{86d(tps)^2 SnC_w}{10^8} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For square poles the corresponding form is—

$$V_r = \frac{93d(tps)^2 SnC_w}{10^8} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The use of the above formulæ may be extended by introducing the total watts with which the machine is dealing. It is necessary to discriminate between two-circuit and multiple-circuit windings, because of the difference in the value of C_w in the two cases.

A. *Multiple-circuit Windings.*

In this case, since—

$$E = \text{flux per pole} \times \text{armature conductors} \times n \times 10^{-8}$$

and since conductors $= 2 \times$ armature-turns $= 2S \times tps$

and $C = C_w \times p$

we have $EC = 2 \times \text{flux per pole} \times Sn(tps)C_w p \times 10^{-8}$

Now, from equation (1) above—

$$2Sn(tps)C_w 10^{-8} = V_r/(40l + 6pp)(tps)$$

whence

$$EC = \frac{\text{flux per pole} \times p \times V_r}{(40l + 6pp)(tps)}$$

or

$$V_r = \text{watts}\,\frac{(40l + 6pp)(tps)}{\text{flux per pole} \times \text{poles}} \quad . \quad . \quad . \quad . \quad (5)$$

Substituting from equation (2) above, we get in the case of circular poles—

$$V_r = \text{watts}\,\frac{43d\,.\,(tps)}{\text{flux per pole} \times \text{poles}} \quad . \quad . \quad . \quad . \quad (6)$$

and for square poles—

$$V_r = \text{watts}\,\frac{46{\cdot}5d\,.\,(tps)}{\text{flux per pole} \times \text{poles}} \text{ approximately} \quad . \quad . \quad (7)$$

If we assume a magnetic density in the pole of 10^5 turns per sq. in., we get for circular poles—

$V_r = 0{\cdot}55$ kilowatt per pole $\times$ turns per section $\div$ pole-diameter (8)

and for square poles—

$V_r = 0{\cdot}465$ kilowatt per pole $\times$ turns per section $\div$ length of pole side . . . . (9)

B. *Two-circuit Windings.*

In the case of multiple-circuit windings the number of turns lying between two neighbouring commutator segments is simply the number of turns per section. In two-circuit windings, however, the number of turns between two neighbouring segments is $p/2$ times the number of turns per section. Also, since selective commutation is liable to take place in two-circuit windings when as many sets of brushes are used as there are poles, it is advisable to assume the use of only two sets of brushes.

Then the number of turns short circuited per segment $= (tps) \times p/2$; so that V_r for a two-circuit winding is under these conditions $p/2$ times V_r for a multiple-circuit winding having the same number of turns per section.

APPENDIX VII.

WIRE TABLES.

Number S.W.G.	Diameter, inches.	Area, sq. inches.	Weight, lbs. per 1000 yds.	Resistance (ohms) per 1000 yards.		Number S.W.G.
				Cold = 15° C.	Hot = 60° C.	
30	0·0124	0·000121	1·4	199·1	233·4	30
29	0·0136	0·000145	1·676	165·5	194·4	29
28	0·0148	0·000172	1·989	139·8	163·8	28
27	0·0164	0·000211	2·440	113·8	133·5	27
26	0·018	0·000254	2·942	94·48	111	26
25	0·020	0·000314	3·633	76·53	90	25
24	0·022	0·000380	4·392	63·24	74·4	24
23	0·024	0·000452	5·233	53·13	62·1	23
22	0·028	0·000616	7·120	39·05	45·9	22
21	0·032	0·000804	9·301	29·90	35·1	21
20	0·036	0·001018	11·77	23·62	27·7	20
19	0·040	0·001257	14·53	19·13	22·4	19
18	0·048	0·001810	20·93	13·28	15·6	18
17	0·056	0·002463	28·48	9·762	11·5	17
16	0·064	0·003217	37·20	7·478	8·8	16
15	0·072	0·004072	47·09	5·904	6·95	15
14	0·080	0·005027	58·13	4·784	5·6	14
13	0·092	0·006648	76·88	3·617	4·2	13
12	0·104	0·008495	98·24	2·831	3·33	12
11	0·116	0·01057	122·2	2·275	2·66	11
10	0·128	0·01287	148·8	1·868	2·18	10
9	0·144	0·01629	188·4	1·476	1·73	9
8	0·160	0·02011	232·5	1·195	1·4	8
7	0·176	0·02433	281·3	0·9881	1·16	7
6	0·192	0·2895	334·7	0·8307	0·972	6
5	0·212	0·03530	408·20	0·6813	0·798	5
4	0·232	0·04227	488·80	0·5688	0·666	4
3	0·252	0·04988	576·70	0·4821	0·567	3
2	0·276	0·05983	692·00	0·4019	0·48	2
1	0·300	0·07069	817·60	0·3402	0·4	1
1/0	0·324	0·08245	953·40	0·2917	0·342	1/0
2/0	0·348	0·09511	1099	0·2528	0·3	2/0
3/0	0·372	0·1087	1257	0·2212	0·26	3/0
4/0	0·400	0·1257	1453	0·1913	0·224	4/0

APPENDIX VIII.

Specific Resistance of Copper at Various Temperatures.

Temp. degrees C.	Resistance per inch cube in ohms.
0	63×10^{-8}
10	65×10^{-8}
15	67×10^{-8}
20	68×10^{-8}
30	71×10^{-8}
40	73×10^{-8}
50	76×10^{-8}
60	78×10^{-8}
70	81×10^{-8}
80	84×10^{-8}

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